EXAM 3, MATH 233
THURSDAY, APRIL 10, 2003

This examination has 20 multiple choice questions, and two essay questions. Please check it over and if you find it to be incomplete, notify the proctor. Do all your supporting calculations in this booklet. In case of a doubtful mark on your answer card, we can then check here. When you mark your card, use a soft lead pencil (#2). Erase fully any answers you want to change. Problems 1 through 20 are worth one point apiece.

On problems 21 and 22, show all your work and indicate clearly your answer to the problem. Partial credit will be given for partially completed solutions. Each of these problems is worth 5 points.

There is a total of 30 points for the whole examination.

You may use a scientific calculator.

(1) The function $f(x,y) = x^3y + 12x^2 - 8y$ has one critical point (a,b). Compute $f(a,b)$ and say whether it is a local maximum, local minimum or a saddle point.

(A) -24, local minimum.
(B) 24, local maximum.
(C) 48, saddle point.
(D) 48, local maximum.
(E) -48, local minimum.
(F) 0, saddle point.
(G) 0, local minimum.
(H) 32, local maximum.
(I) -32, local minimum.
(J) 32, saddle point.

$f_x = 3x^2y + 24x$, $f_y = x^3 - 8$.

At (a,b), $f_y(a,b) = 0 \Rightarrow a^3 = 8 \Rightarrow a = 2$.

$f_x(a,b) = 0 \Rightarrow 12b + 48 = 0 \Rightarrow b = -4$.

So $(a,b) = (2,-4)$.

$$D = \begin{vmatrix} f_{xx}(a,b) & f_{xy}(a,b) \\ f_{xy}(a,b) & f_{yy}(a,b) \end{vmatrix}.$$

$f_{xx} = 6xy + 24$, $f_{xy} = 3x^2$, $f_{yy} = 0$.

$$D(a,b) = \begin{vmatrix} -24 & 12 \\ 12 & 0 \end{vmatrix} = -144.$$

$D < 0$.

So (a,b) is a saddle point

$f(a,b) = -32 + 48 + 32 = 48$

(2) The maximum value of the function $V = xyz$, $x, y, z \geq 0$ with $x + y + z = 30$ is,

(A) 300
(B) 400
(C) 500
(D) 600
(E) 700
(F) 800
(G) 900
(H) 1000
(I) 1100
(J) 1200

Since $z = 30 - x - y$,

$V = xy(30 - x - y)$.

$V_x = 30y - 2xy - y^2$

$V_y = 30x - x^2 - 2xy$

$V_x = 0 = V_y$ implies, since $x = 0$ or $y = 0$ makes $V = 0$, we may assume $x \neq 0$ and $y \neq 0$.

$\therefore \quad 30 = 2x + y$ and $30 = x + 2y$.

$\therefore \quad 2x + y = x + 2y \Rightarrow x = y \quad \therefore \quad 3x = 30$

$\therefore \quad x = 10, y = 10$ and then $z = 10$.

$\therefore \quad V = 10^3 = \boxed{1000}$

(3) The minimum and maximum values of the function $f(x,y) = x^2 + y^2$ with the constraint $x^4 + y^4 = 1$ are,

(A) $0, 2$
(B) $0, \sqrt{2}$
(C) $1, \sqrt{2}$
(D) $0, \sqrt{5}$
(E) $1, \sqrt{5}$
(F) $\sqrt{2}, \sqrt{5}$
(G) $0, 1$
(H) $\sqrt{2}, \sqrt{2}+1$
(I) The minimum is 0 and maximum does not exist.
(J) The maximum is $\sqrt{5}$ and minimum does not exist.

Using Lagrange multipliers, we must solve:

$f_x = \lambda g_x$, $f_y = \lambda g_y$, $g = 1$ where $g(x,y) = x^4 + y^4$.

$\therefore \quad 2x = \lambda 4x^3$, $2y = \lambda 4y^3 \Rightarrow x = 2\lambda x^3$, $y = 2\lambda y^3$.

$\therefore \quad x^2 + y^2 = 2\lambda(x^4 + y^4) = \boxed{2\lambda}$.

If $x = 0$, then $y = \pm 1$, and thus, $2\lambda y^2 = 1 \Rightarrow 2\lambda = 1$.

Similarly, if $y = 0$, then $x = \pm 1$ and then $2\lambda x^2 = 1 \Rightarrow 2\lambda = 1$.

Finally, if $x \neq 0$ & $y \neq 0$, then $2\lambda x^2 = 1$, $2\lambda y^2 = 1$

and so, $\Rightarrow x^2 = y^2 = \frac{1}{2\lambda}$. $x^4 + y^4 = 1$ implies,

$\left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 = 1 \Rightarrow \lambda^2 = \frac{1}{2}$. Since $x^2 = \frac{1}{2\lambda}$, $\lambda > 0$.

$\therefore \quad \lambda = \frac{1}{\sqrt{2}}$. $\therefore \quad 2\lambda = \frac{2}{\sqrt{2}} = \sqrt{2}$.

So the possible values are 1 and $\sqrt{2}$.

Thus the maximum is $\sqrt{2}$ and minimum is 1.

(4) The maximum and minimum values of the function $f(x, y, z) = x + y$, subject to the constraint $x^2 + y^2 + z^2 = 4$, are

(A) $2, -2$
(B) $2\sqrt{2}, -2\sqrt{2}$
(C) $2 + \sqrt{2}, 2 - \sqrt{2}$
(D) $1 + \sqrt{2}, 1 - \sqrt{2}$
(E) $1 + 2\sqrt{2}, 1 - 2\sqrt{2}$
(F) $2 + \sqrt{3}, 2 - \sqrt{3}$
(G) $1 + \sqrt{3}, 1 - \sqrt{3}$
(H) $1 + 2\sqrt{3}, 1 - 2\sqrt{3}$
(I) $2\sqrt{3}, -2\sqrt{3}$
(J) The maximum does not exist and the minimum is $-2\sqrt{3}$.

Taking $g(x,y,z) = x^2 + y^2 + z^2$,

we have $1 = \frac{\partial f}{\partial x} = \lambda \cdot \frac{\partial g}{\partial x} = 2\lambda x$

$1 = \frac{\partial f}{\partial y} = \lambda \cdot \frac{\partial g}{\partial y} = 2\lambda y$

$0 = \frac{\partial f}{\partial z} = \lambda \frac{\partial g}{\partial z} = 2\lambda z$. and $g = 4$.

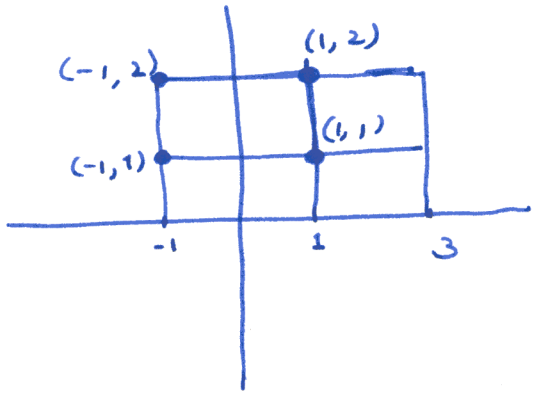
The first says that $\lambda \neq 0$ and so from the last we get $z = 0$. The first two says $x = y$.

$g = 4$ then says, $x^2 + x^2 + 0 = 4 \Rightarrow x = \pm\sqrt{2}$.

$\therefore f(x,y,z) = 2\sqrt{2}$ or $-2\sqrt{2}$. Thus the maximum is $2\sqrt{2}$ and minimum is $-2\sqrt{2}$.

(5) Let R be the rectangle $[-1,3] \times [0,2]$. We divide $[-1,3]$ into two intervals of equal length and divide $[0,2]$ into two intervals of equal length. Estimate the value of $\int\int_R 2x^2 - y^2 dA$, where the sample points are taken to be the upper lefthand corner points of the subrectangles, by the Riemann sum associated with the choice of the sample points and the subrectangles.

(A) -1
(B) -2
(C) -3
(D) -4
(E) -5
(F) 6
(G) 7
(H) 8
(I) 9
(J) 0



The Riemann sum is:

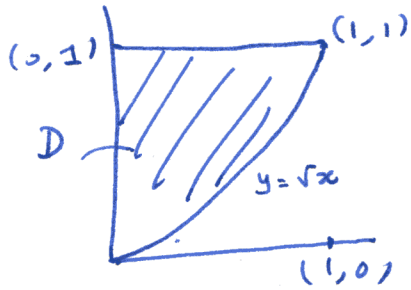
$$[2(-1)^2 - 1] \cdot 2 + [2(1)^2 - 1] \cdot 2 + [2(-1)^2 - 4] \cdot 2 + [2(1)^2 - 4] \cdot 2$$

$$= 2\{1 + 1 - 2 - 2\} = \boxed{-4}$$

(6) By interchanging the order of integration, evaluate the double integral:

$$\iint_D \frac{3}{4+y^3}\,dA$$

over the region $D = \{(x,y) : 0 \le x \le 1,\ \sqrt{x} \le y \le 1\}$



(A) 1/20
(B) -1/20
(C) 9/400
(D) -9/400
(E) $\ln 5$
(F) $\ln \frac{5}{4}$
(G) $-\ln \frac{5}{4}$
(H) $e^{5/4}$
(I) $-e^{5/4}$
(J) $-\ln 5$

$$\iint_D \frac{3}{4+y^3}\,dA = \int_0^1 \int_0^{y^2} \frac{3}{4+y^3}\,dx\,dy$$

$$= \int_0^1 \frac{3x}{4+y^3}\Big|_0^{y^2}\,dy$$

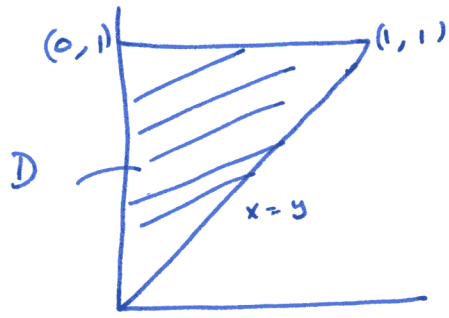
$$= \int_0^1 \frac{3y^2}{4+y^3}\,dy = \ln(4+y^3)\Big|_0^1 = \ln 5 - \ln 4$$

$$= \boxed{\ln \frac{5}{4}}$$

(7) By interchanging the order of integration, compute the iterated integral

$$\int_0^1 \int_y^1 e^{x^2}\, dx\, dy.$$

(A) 1/2
(B) -1/2
(C) 1
(D) -1
(E) $e/2 - 1/2$
(F) $e/2 + 1/2$
(G) $e^2 - 1$
(H) $2e - 1$
(I) $2e^2 - 1$
(J) $2e^2 - 2$



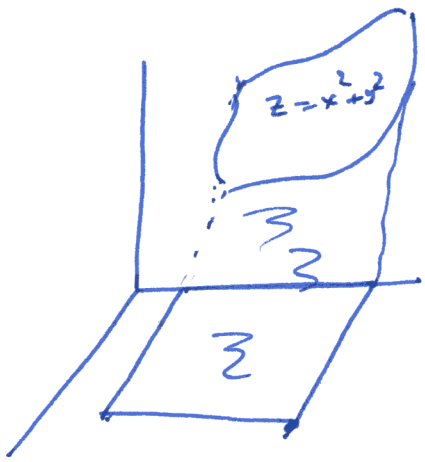
$$\int_0^1 \int_y^1 e^{x^2} dx\, dy = \int_0^1 \int_0^x e^{x^2} dy\, dx$$

$$= \int_0^1 e^{x^2} \cdot y \Big|_{y=0}^{y=x} dx$$

$$= \int_0^1 e^{x^2} \cdot x\, dx = \frac{e^{x^2}}{2} \Big|_0^1 = \boxed{\frac{e}{2} - \frac{1}{2}}$$

(8) Find the volume of the solid lying under the paraboloid $z = x^2 + y^2$ and above the rectangle $[0, 3] \times [1, 4]$.

(A) 36
(B) 72
(C) 18
(D) 90
(E) 25/2
(F) 96
(G) 37/2
(H) 18
(I) 44
(J) 15/2



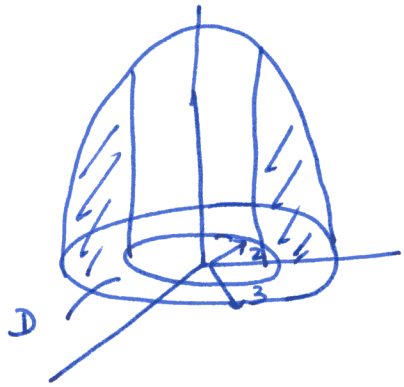
$$\int_1^4 \int_0^3 (x^2+y^2)\, dx\, dy = \int_1^4 \frac{x^3}{3} + y^2 x \Big|_0^3 \, dy$$

$$= \int_1^4 (9 + 3y^2)\, dy = 9y + y^3 \Big|_1^4$$

$$= (36 + 64) - (9 + 1) = 90$$

(9) Find the volume of the solid inside the paraboloid $z = 9 - x^2 - y^2$, outside the cylinder $x^2 + y^2 = 4$ and above the $xy-$plane.

(A) $25\pi/2$
(B) 25π
(C) $35\pi/2$
(D) 35π
(E) 50π
(F) $55\pi/2$
(G) $65\pi/2$
(H) 65π
(I) $67\pi/2$
(J) $15\pi/2$



Volume $= \iint_D (9 - x^2 - y^2)\, dA$, where

D is the annulus, the region between the circles of radius 3 and 2 on the xy-plane.

Changing into polar co-ordinates, $x = r\cos\theta$, $y = r\sin\theta$.

we see that $D: \{ 2 \le r \le 3;\ 0 \le \theta \le 2\pi \}$.

$$\therefore \text{Volume} = \int_2^3 \int_0^{2\pi} (9 - r^2) r\, d\theta\, dr = 2\pi \int_2^3 (9 - r^2)\, r\, dr$$

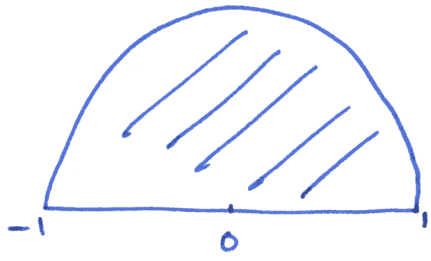
$$= 2\pi \left(\frac{9r^2}{2} - \frac{r^4}{4} \right)\Big|_2^3 = 2\pi \left[\left(\frac{81}{2} - \frac{81}{4} \right) - (18 - 4) \right]$$

$$= 2\pi \left[\frac{81}{4} - 14 \right] = \frac{\pi}{2} [81 - 56] = \frac{25\pi}{2}.$$

(10) Evaluate the iterated integral:

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} x^2\,(x^2+y^2)^2\,dy\,dx$$

(A) $25/7$
(B) $3\sqrt{2}/2$
(C) $2\sqrt{3}/3$
(D) $2\sqrt{5}/3$
(E) $2\sqrt{5}$
(F) $\pi/4$
(G) $3\pi/16$
(H) $3\pi/4$
(I) $\pi/8$
(J) $\pi/16$



change into polar co-ordinates.

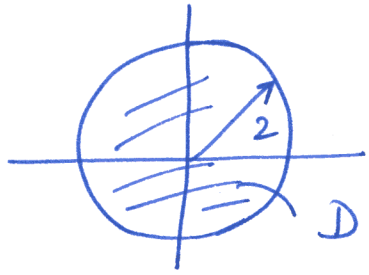
So $\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} x^2(x^2+y^2)^2\,dy\,dx = \int_{0}^{\pi}\int_{0}^{1} r^2\cos^2\theta \cdot r^4 \cdot r\,dr\,d\theta$

$= \int_{0}^{\pi}\cos^2\theta\,d\theta \cdot \int_{0}^{1} r^7\,dr = \int_{0}^{\pi}\frac{1+\cos 2\theta}{2}\,d\theta \cdot \frac{r^8}{8}\Big|_0^1$

$= \frac{1}{8}\cdot\left[\frac{\theta}{2}+\frac{\sin 2\theta}{4}\right]_0^{\pi} = \frac{\pi}{16}$.

(11) Electric charge is distributed over the disk $x^2 + y^2 \leq 4$ so that the charge density at (x, y) is $x^2 + y^2$ (measured in coulombs per square meter). Find the total charge on the disk.

(A) π
(B) 2π
(C) 3π
(D) 4π
(E) 5π
(F) 6π
(G) 7π
(H) 8π
(I) 9π
(J) 10π



Total charge $= \iint_D (x^2+y^2)\, dA$

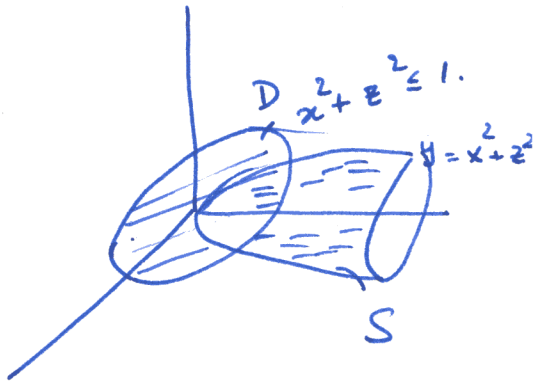
Change into polar co-ordinates.

$$= \int_0^{2\pi} \int_0^2 r^2 \cdot r \cdot dr\, d\theta = 2\pi \int_0^2 r^3 dr = 2\pi \left. \frac{r^4}{4} \right|_0^2$$

$$= 8\pi.$$

(12) Find the area of the surface $y = x^2 + z^2$ that lies inside the cylinder $x^2 + z^2 = 1$.

(A) π
(B) $\pi/2$
(C) 2π
(D) $\pi\sqrt{3}/2$
(E) $1 + \pi 2^{3/2}$
(F) $\pi(5^{3/2} - 1)/6$
(G) 3π
(H) $3\pi/2$
(I) $\pi(1 + \sqrt{2})$
(J) 4π



$A(S)$ = area of S

$$= \iint_D \sqrt{1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2}\, dA$$

$$= \iint_D \sqrt{1 + 4x^2 + 4z^2}\, dA.$$ change into polar co-ordinates

$$= \int_0^{2\pi} \int_0^1 \sqrt{1+4r^2}\; r\, dr\, d\theta$$

$$= 2\pi \cdot \int_0^1 \sqrt{1+4r^2}\; r\, dr = 2\pi \cdot \left. (1+4r^2)^{3/2} \cdot \frac{2}{3} \cdot \frac{1}{8} \right|_0^1$$

$$= \frac{\pi}{6} \left[5^{3/2} - 1 \right].$$

(13) Evaluate $\int_0^1 \int_0^x \int_0^y xyzdzdydx$.

(A) 1/40
(B) 1/42
(C) 1/44
(D) 1/46
(E) 1/48
(F) 1/50
(G) 1/52
(H) 1/54
(I) 1/56
(J) 1/58

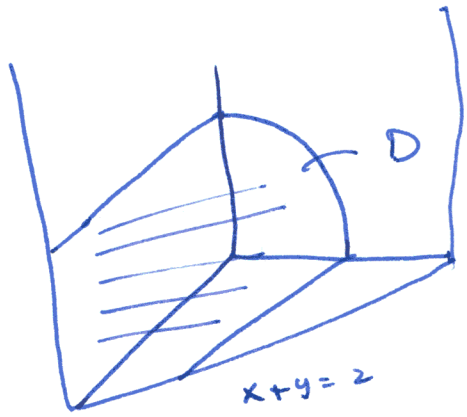
$$\int_0^1 \int_0^x \int_0^y xyz\,dz\,dy\,dx = \int_0^1 \int_0^x \frac{xyz^2}{2}\Big|_{z=0}^{y} dy\,dx$$

$$= \int_0^1 \int_0^x \frac{xy^3}{2}\,dy\,dx = \int_0^1 \frac{xy^4}{8}\Big|_{y=0}^{x} dx = \int_0^1 \frac{x^5}{8}\,dx$$

$$= \frac{x^6}{48}\Big|_0^1 = \frac{1}{48}.$$

(14) Find the volume bounded by the planes $y = 0, z = 0, x + y = 2$ and the cylinder $y^2 + z^2 = 1$ in the first octant.

(A) $\pi/2$
(B) 1
(C) $(3\pi - 2)/6$
(D) 2
(E) $(\pi + 4)/2$
(F) 3
(G) 4
(H) 5
(I) 6
(J) π



Volume $= \iint_D (2-y)\, dA$. Change into polar coord.

$$= \int_0^{\pi/2} \int_0^1 (2 - r\cos\theta)\, r\, dr\, d\theta$$

$$= \frac{\pi}{2} \cdot r^2 \Big|_0^1 - \int_0^{\pi/2} \int_0^1 r^2 \cos\theta\, dr\, d\theta$$

$$= \frac{\pi}{2} - \int_0^{\pi/2} \cos\theta\, d\theta \cdot \int_0^1 r^2\, dr$$

$$= \frac{\pi}{2} - \sin\theta \Big|_0^{\pi/2} \cdot \frac{r^3}{3}\Big|_0^1 = \frac{\pi}{2} - \frac{1}{3} = \frac{3\pi - 2}{6}.$$

(15) Find the x-coordinate of the center of mass of the square $0 \le x \le 1, 0 \le y \le 1$, where the density is $\rho(x, y) = y$.

(A) 0.1
(B) 0.2
(C) 0.3
(D) 0.4
(E) 0.5
(F) 0.6
(G) 0.7
(H) 0.8
(I) 0.9
(J) 1

$$m = \iint_D \rho(x,y)\, dA = \int_0^1 \int_0^1 y \, dx \, dy$$

$$= \int_0^1 dx \int_0^1 y \, dx = \frac{y^2}{2}\Big|_0^1 = \frac{1}{2}.$$

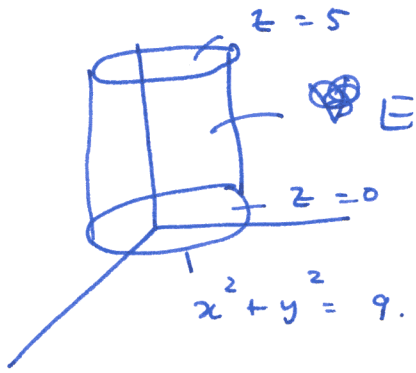
$$\bar{x} = \frac{1}{m} \iint_D x \rho(x,y)\, dA = 2 \int_0^1 \int_0^1 xy \, dx \, dy$$

$$= 2 \int_0^1 x \, dx \int_0^1 y \, dx = 2 \cdot \frac{x^2}{2}\Big|_0^1 \cdot \frac{y^2}{2}\Big|_0^1 = 2 \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{2} = 0.5$$

(16) Evaluate $\int\int\int_E \sqrt{x^2+y^2}\,dV$, where E is the region that lies inside the cylinder $x^2+y^2=9$ and between the planes $z=0$ and $z=5$.

(A) 6.
(B) 6π.
(C) 18.
(D) 18π
(E) 30.
(F) 30π.
(G) 45.
(H) 45π
(I) 90.
(J) 90π.



E : $\{ x^2+y^2 \le 9,\ 0 \le z \le 5 \}$.

$\iiint_E \sqrt{x^2+y^2}\,dV$: Change into cylindrical co-ord.

E : $\{ 0 \le r \le 3,\ 0 \le \theta \le 2\pi,\ 0 \le z \le 5 \}$.

$$\therefore = \int_0^3 \int_0^{2\pi} \int_0^5 r \cdot r \, dz\, d\theta\, dr$$

$$= \int_0^3 r^2 dr \cdot \int_0^{2\pi} d\theta \cdot \int_0^5 dz = \frac{r^3}{3}\Big|_0^3 \cdot 2\pi \times 5$$

$$= 9 \times 2\pi \times 5 = 90\pi.$$

(17) Evaluate $\int\int\int_E x^2+y^2+z^2\,dV$, where E is the ball $x^2+y^2+z^2 \le 1$.

(A) $\frac{2}{5}$.
(B) $\frac{4\pi}{5}$.
(C) $\frac{1}{2}$.
(D) $\frac{\pi}{2}$.
(E) $\frac{2}{3}$.
(F) $\frac{2\pi}{3}$.
(G) 1.
(H) π.
(I) 2.
(J) 2π.

Change into spherical co-ordinates.

$E : \{ 0 \le \rho \le 1,\ 0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi \}$.

$$\therefore = \int_0^1 \int_0^{2\pi} \int_0^{\pi} \rho^2 \cdot \rho^2 \sin\phi \cdot d\phi\, d\theta\, d\rho$$

$$= \int_0^1 \rho^4 d\rho \cdot \int_0^{2\pi} d\theta \cdot \int_0^{\pi} \sin\phi\, d\phi$$

$$= \frac{\rho^5}{5}\Big|_0^1 \cdot 2\pi \cdot -\cos\phi\Big|_0^{\pi}$$

$$= \frac{1}{5} \times 2\pi \times 2 = \frac{4\pi}{5}.$$

(18) Find the Jacobian of the transformation given by $x = u^2 - v^2$ and $y = u^2 + v^2$.

(A) $\frac{uv}{(u^2-v^2)}$.
(B) $-\frac{uv}{(u^2-v^2)}$
(C) $\frac{2uv}{(u^2-v^2)}$
(D) $8uv$
(E) $\frac{uv}{(u^2-v^2)^2}$
(F) $u^2 + v^2$
(G) $\frac{2uv}{(u^2-v^2)^2}$
(H) $-\frac{2uv}{(u^2-v^2)^2}$
(I) $\frac{2uv}{(u^2-v^2)^4}$
(J) $u^2 - v^2$.

Jacobian, $\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$

$= \begin{vmatrix} 2u & -2v \\ 2u & 2v \end{vmatrix} = 4uv + 4uv = 8uv$

(19) Find the Jacobian of the transformation given by $x = uv$, $y = vw$, $z = uw$.

(A) uvw.
(B) $-uvw$.
(C) $2uvw$.
(D) $-2uvw$.
(E) u^2vw.
(F) $-u^2vw$.
(G) $2u^2vw$.
(H) $-2u^2vw$.
(I) $2u^2v^2w^2$.
(J) 0.

$$\text{Jacobian} = \frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

$$= \begin{vmatrix} v & u & 0 \\ 0 & w & v \\ w & 0 & u \end{vmatrix} = v \begin{vmatrix} w & v \\ 0 & u \end{vmatrix} - u \begin{vmatrix} 0 & v \\ w & u \end{vmatrix} + 0 \begin{vmatrix} 0 & w \\ w & 0 \end{vmatrix}$$

$$= v \cdot uw - u(-vw)$$

$$= 2uvw.$$

(20) Evaluate $\int\int_R 10(2x-y)\,dA$, where R is the region bounded by the lines $2x-y=1$, $2x-y=-3$, $3x+y=1$, and $3x+y=-2$. (Hint: Use the transformation $u=2x-y$ and $v=3x+y$. Find x and y in terms of u and v when calculating the Jacobian.)

(A) $\frac{3}{2}$.
(B) $-\frac{3}{2}$.
(C) -12.
(D) $-\frac{1}{2}\sin 1$.
(E) $\frac{3}{2}\sin 1$.
(F) -24.
(G) $\frac{3}{2}\sin 2$.
(H) $-\frac{3}{2}\sin 2$.
(I) -48.
(J) $-2\sin 2$.

$u = 2x-y,\ v = 3x+y$

$\therefore\ u+v = 5x$ or $x = \frac{1}{5}(u+v)$.

$y = 2x-u = \frac{2}{5}(u+v)-u = -\frac{3}{5}u+\frac{2}{5}v$.

$\therefore\ \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{5} & \frac{1}{5} \\ -3/5 & \frac{2}{5} \end{vmatrix} = \frac{2}{25}+\frac{3}{25} = \frac{1}{5}$.

$$\iint_R 10(2x-y)\,dA = \int_{u=-3}^{1}\int_{v=-2}^{1} 10u\cdot\frac{1}{5}\,du\,dv$$

$$= \int_{u=-3}^{1} 2u\,du \cdot \int_{v=-2}^{1} dv = u^2\Big|_{-3}^{1}\cdot v\Big|_{-2}^{1}$$

$$= (1-9)\cdot(1+2) = -24.$$

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(21) The density function of a ~~square~~ lamina is

$$f(x,y) = Cx(1+y) \quad \text{if } 0 \le x \le 1,\ 0 \le y \le 2$$

for some constant C.

a. What is C if the total mass of the material is 1?

b. Find the y-coordinate of the center of mass of the material.

$$1 = m = \int_0^1 \int_0^2 C\,x(1+y)\,dy\,dx = C\int_0^1 x\,dx \cdot \int_0^2 (1+y)\,dy$$

$$= C \cdot \frac{x^2}{2}\Big|_0^1 \cdot \left. y + \frac{y^2}{2}\right|_0^2 = C \cdot \frac{1}{2} \cdot (2+2) = 2C.$$

$$\therefore \boxed{C = \frac{1}{2}}$$

$$\bar{y} = \frac{1}{m} \iint_D y\, f(x,y)\, dA$$

$$= \int_0^1 \int_0^2 \frac{xy(1+y)}{2}\,dy\,dx = \frac{1}{2}\int_0^1 x\,dx \cdot \int_0^2 (y+y^2)\,dy$$

$$= \frac{1}{2} \cdot \frac{x^2}{2}\Big|_0^1 \cdot \left. \frac{y^2}{2} + \frac{y^3}{3}\right|_0^2 = \frac{1}{2} \times \frac{1}{2} \times \left(2 + \frac{8}{3}\right).$$

$$= \frac{1}{4} \cdot \frac{14}{3} = \boxed{\frac{7}{6}}$$

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(22) Find the area of the part of the plane with vector equation

$$\mathbf{r}(u,v) = < 1+v, u-2v, 3-5u+v >$$

that is given by $0 \le u \le 1, 0 \le v \le 1$.

$A(S)$ = area of the surface

$$= \iint_D \left| \vec{r}_u \times \vec{r}_v \right| dA, \quad \text{where } D: \{0 \le u \le 1, \ 0 \le v \le 1\}.$$

$$\vec{r}_u = \langle 0, 1, -5 \rangle$$

$$\vec{r}_v = \langle 1 \quad -2 \quad 1 \rangle.$$

$$\therefore \vec{r}_u \times \vec{r}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -5 \\ 1 & -2 & 1 \end{vmatrix} = \vec{i}(1-10) - \vec{j}(5) + \vec{k}(-1).$$

$$= 9\vec{i} - 5\vec{j} - \vec{k}.$$

$$\therefore \left| \vec{r}_u \times \vec{r}_v \right| = \sqrt{9^2+5^2+1} = \sqrt{81+25+1}$$

$$= \sqrt{107}$$

$$\therefore A(S) = \int_0^1 \int_0^1 \sqrt{107}\, dx\, dy = \sqrt{107}$$