Math 233 S. G. Krantz
Spring, 2009 February 4, 2009

First Midterm Exam

NAME:

STUDENT ID NUMBER:

General Instructions: Read the instructions to each question carefully. Do precisely what is asked—nothing more and nothing less. Note that the first fifteen questions are multiple choice and the last two are short answer. This means that, for the first fifteen questions, you indicate your answer by filling in a bubble with a pencil on the card provided. For the last two questions you actually write your answer *on this test paper*. *REMEMBER:* No calculators!! You may have a 3×5 card with notes and formulas.

Each of the 17 questions on this exam is worth 5 points, for a total of 85 points.

Ask questions of the proctors or the professor if anything is unclear.

1. The sum of the vectors $\mathbf{v} = \langle 2, 4, 9 \rangle$ and $\mathbf{w} = \langle -1, 6, -2 \rangle$ is

(a) $\langle 1, 10, 7 \rangle$ **(b)** $\langle 2, 8, 12 \rangle$ **(c)** $\langle 1, -10, 7 \rangle$ **(d)** $\langle -1, 10, 7 \rangle$
(e) $\langle 1, 10, -7 \rangle$

2. If $\mathbf{u} = \langle 4, 1, 1\rangle$, $\mathbf{v} = \langle 2, 1, 4\rangle$, and $\mathbf{w} = \langle -6, 2, 3\rangle$, then $\mathbf{u} \times \mathbf{w} - \mathbf{v} =$

(a) $\langle 1, 1, 1\rangle$ **(b)** $\langle 2, 18, 3\rangle$ **(c)** $\langle -1, -19, 10\rangle$ $\langle -1, 19, 20\rangle$
(d) $\langle 2, 2, 3\rangle$ **(e)** $\langle 1, 19, -10\rangle$

3. If $\mathbf{u} = 3\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 6\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$ then $\mathbf{u} \cdot \mathbf{v} =$

(a) 25 **(b)** 42 **(c)** 21 **(d)** -65 **(e)** 38

4. If $\mathbf{u} = \langle 2, 2, 6\rangle$ and $\mathbf{v} = \langle -4, 1, 0\rangle$ then $P_{\mathbf{u}}(\mathbf{v})$ and $P_{\mathbf{v}}(\mathbf{u})$ equal

(a) $\langle 2, 1, 1\rangle$, $\langle 1, 2, 2\rangle$ **(b)** $\left\langle -\frac{3}{11}, -\frac{3}{11}, -\frac{9}{11}\right\rangle$, $\left\langle \frac{24}{17}, -\frac{6}{17}, 0\right\rangle$
(c) $\langle 1, 0, 1\rangle$, $\langle 1, 5, 7\rangle$ **(d)** $\left\langle \frac{36}{25}, \frac{48}{25}, 0\right\rangle$, $\left\langle \frac{48}{25}, \frac{3}{25}, 0\right\rangle$
(e) $\langle 2, 1, 5\rangle$, $\langle 5, 1, 2\rangle$

5. A unit vector that is perpendicular to both $\mathbf{u} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = \mathbf{i} - \mathbf{j} + 5\mathbf{k}$ is given by

(a) $\left\langle -\frac{18}{\sqrt{494}}, -\frac{13}{\sqrt{494}}, \frac{1}{\sqrt{494}}\right\rangle$ **(b)** $\left\langle \frac{8}{\sqrt{394}}, \frac{10}{\sqrt{394}}, \frac{-1}{\sqrt{394}}\right\rangle$ **(c)** $\langle 1, 2, 2\rangle$
(d) $\langle 1/\sqrt{2}, -1/\sqrt{2}, 9\rangle$ **(e)** $\left\langle \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\rangle$

6. The equation of the plane passing through the point $(-3, 4, 7)$ and having normal vector $\mathbf{n} = \langle -3, 1, 5\rangle$ is

(a) $2x - 7y + 4z = 6$ **(b)** $3x - y - 5z = 48$ **(c)** $x - 3y + 5z = 24$
(d) $-3x + y + 5z = 48$ **(e)** $3x + y - 5z = 48$

7. The equation of the plane passing through the points $P = (2, 2, 4)$, $Q = (-3, 1, 1)$, and $R = (4, 3, 3)$ is

(a) $3x - 4y + 7z = 6$ **(b)** $4x - 11y - 3z = -26$
(c) $-3x - 4y + 7z = 26$ **(d)** $4x + 11y + 3z = 26$
(e) $x + y + 3z = 26$

8. The parametric equations for the line passing through $P = (2, 1, 0)$ and having direction vector $\mathbf{v} = \langle 4, 1, 2 \rangle$ are

(a) $x = 3t - 1,\ y = 2t + 2,\ z = t + 1$
(b) $x = 4t + 6,\ y = t - 5,\ z = -t + 1$
(c) $x = 4t + 2,\ y = t + 1,\ z = 2t$
(d) $x = 4t - 2,\ y = t - 1,\ z = 3t$
(e) $x = 4t - 4,\ y = -t,\ z = 2t - 5$

9. The lines

$$x = 2t - 5, y = t + 1, z = -t + 4$$

and

$$x = t + 1, y = -t + 3, z = 6t + 1$$

(a) intersect at a single point **(b)** intersect at three points
(c) intersect at no points **(d)** intersect at all points
(e) intersect at some points

10. The line

$$x = 2t - 7, y = t + 4, z = -t + 6$$

can be represented as the intersection of two planes. These planes are

(a) $2x - 3y + z = 1$, $x + y + z = 4$

(b) $x - y + z = 1$, $x + 2y - z = 4$

(c) $2x - 4y + z = 0$, $x + y - 3z = 1$

(d) $x + y = 5$, $x - 2z = 3$

(e) $x - 2y = -15$, $y + z = 10$

11. Find the area of the parallelogram determined by the two vectors $\mathbf{v} = \langle 2, 2, 4 \rangle$ and $\mathbf{w} = \langle 1, 1, 1 \rangle$.

(a) $2\sqrt{2}$ **(b)** 6 **(c)** $3\sqrt{2}$ **(d)** $\sqrt{2}$ **(e)** 1

12. The first and second derivatives of the vector function $\mathbf{r}(t) = t^2\mathbf{i} - (\cos t)\mathbf{j} + (\ln t)\mathbf{k}$ are

(a) $\mathbf{r}'(t) = 2t\mathbf{i} + (\sin t)\mathbf{j} + (1/t)\mathbf{k}$, $\mathbf{r}''(t) = 2\mathbf{i} + (\cos t)\mathbf{j} - (1/t^2)\mathbf{k}$

(b) $\mathbf{r}'(t) = t\mathbf{i} - 2t\mathbf{j} + 3t\mathbf{k}$, $\mathbf{r}''(t) = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$

(c) $\mathbf{r}'(t) = t^2\mathbf{i} + t^3\mathbf{j} + t^4\mathbf{k}$, $\mathbf{r}''(t) = 2t\mathbf{i} + 3t^2\mathbf{j} + 4t^3\mathbf{k}$

(d) $\mathbf{r}'(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} - (\tan t)\mathbf{k}$,
$\mathbf{r}''(t) = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} - (\sec^2 t)\mathbf{k}$

(e) $\mathbf{r}'(t) = -t\mathbf{i} - 2t\mathbf{j} - 3t\mathbf{k}$, $\mathbf{r}''(t) = -\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$

13. A moving body has acceleration given by $-16\mathbf{k}$. It has initial velocity $-12\mathbf{j}$ and initial position $-6\mathbf{i}$. The formula for the position $\mathbf{r}(t)$ is given by

(a) $\mathbf{r}(t) = 6t\mathbf{i} - t^2\mathbf{j} - 5t\mathbf{k}$ **(b)** $\mathbf{r}(t) = -6\mathbf{i} - 12t\mathbf{j} - 8t^2\mathbf{k}$

(c) $\mathbf{r}(t) = 6t\mathbf{i} - 12t^2\mathbf{j} + 8t^3\mathbf{k}$ **(d)** $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$

(e) $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t\mathbf{k}$

14. An feeble arrow is shot into the air from a height of 2 feet at a speed of 8 feet per second. The initial angle of the arrow from the horizontal is $\pi/6$. If the acceleration due to gravity is -32 feet per second squared, determine how far the arrow travels horizontally.

(a) $\sqrt{3}$ feet **(b)** $12\sqrt{3}$ feet **(c)** $800 + 99\sqrt{3}$ feet
(d) $2\sqrt{3}$ feet **(e)** $\sqrt{88} + 99$ feet

15. The arc length of the curve $\mathbf{r}(t) = \cos(t^2)\mathbf{i} - \sin(t^2)\mathbf{j} + t^2\mathbf{k}$ for $0 \leq t \leq \sqrt{\pi}$ is

(a) $\sqrt{2}$ **(b)** $\sqrt{3}$ **(c)** 5 **(d)** $\sqrt{2}\pi$ **(e)** $\frac{\sqrt{6}\pi}{4}$

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16. The tangent line to the curve $\mathbf{r}(t) = t\mathbf{i} - t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(1, -1, 1)$ is given (in parametric form) by

17. The curve $\mathbf{r}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + t\mathbf{k}$, reparametrized according to arc length, is given by