Math 233, Final Exam, December 16th

This exam should have 16 questions. All questions carry equal marks.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Choose the answer that is closest to the solution. Mark your answer card with a **PENCIL** by shading in the correct box.

You may use a calculator, but not one that has a graphing function. Make sure that angles are set to **radians**.

1. Let $F : \mathbb{R}^2 \to \mathbb{R}^3$. Let a be a point in $\mathbb{R}^2$. Then the derivative of F at a is:

A) A number.
B) A vector in $\mathbb{R}^2$.
C) A vector in $\mathbb{R}^3$.
D) A vector in $\mathbb{R}^6$.
E) A 2-by-2 matrix.
F) A 2-by-3 matrix.
G) A 3-by-2 matrix.
H) A 3-by-3 matrix.
I) A 5-by-5 matrix
J) A 6-by-6 matrix.

2. Let $C = \{(e^y, y) : 0 \leq y \leq 1\}$. Calculate $\int_C xe^y dx$.

A) 6.0
B) 6.1
C) 6.2
D) 6.3
E) 6.4
F) 6.5
G) 6.6
H) 6.7
I) 6.8
J) 6.9

3. Let $g(x, y, z) = x^2 + 2y^4 + 5z^6$, and let $S = \{(x, y, z) : g(x, y, z) = 4\}$. Let $f(x, y, z)$ be a differentiable function on $\mathbb{R}^3$.

Consider the following 3 statements:

(I) The maximum value of f on S always occurs at a critical point of f.

(II) The maximum value of f on S always occurs at a point where the gradient of f is a non-zero multiple of the gradient of g.

(III) The maximum value of f on S always occurs at a point where the gradient of f is a multiple of the gradient of g.

Then:

A. All three statements are false.
B. Statement (I) is true, the other two are false.
C. Statement (II) is true, the other two are false.
D. Statement (III) is true, the other two are false.
E. Statement (I) is false, the other two are true.
F. Statement (II) is false, the other two are true.
G. Statement (III) is false, the other two are true.
H. All three statements are true.

4. Let

$$x = u/v, \quad y = v/w, \quad z = 2w.$$

Calculate the Jacobian $\dfrac{\partial(x, y, z)}{\partial(u, v, w)}$ at $(u, v, w) = (0.6, 1.0, 2.0)$.

A) 0.1
B) 0.2
C) 0.3
D) 0.4
E) 0.5
F) 0.6
G) 0.7
H) 0.8
I) 0.9
J) 1.0

5. Calculate the length of the cross-product of $2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ with $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$.

A) 8.1
B) 8.2
C) 8.3
D) 8.4
E) 8.5
F) 8.6
G) 8.7
H) 8.8
I) 8.9
J) 9.0

6. Let C be the arc of the parabola $y = 2x^2$ from $(-1, 2)$ to $(1, 2)$. Let $\mathbf{F}(x, y) = 3x^2\mathbf{i} + 3y^2\mathbf{j}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

A) -4
B) -3
C) -2
D) -1
E) 0
F) 1
G) 2
H) 3
I) 4
J) 5

7. Consider the three vector fields

$$F(x,y) = \begin{pmatrix} 2xy - y \\ x^2 - x \end{pmatrix} \qquad G(x,y) = \begin{pmatrix} xy - y \\ x^2 - x \end{pmatrix} \qquad H(x,y,z) = \begin{pmatrix} 2z - y^2 \\ -2xy \\ 2x \end{pmatrix}.$$

Then:

A) All 3 are conservative.
B) Two are conservative, F is not.
C) Two are conservative, G is not.
D) Two are conservative, H is not.
E) Only F is conservative.
F) Only G is conservative.
G) Only H is conservative.
H) None are conservative.

8. Evaluate the line integral

$$\int_C \cos x dx + x \sin y dy$$

where C is the positively oriented ellipse $\{x^2 + 4y^2 = 1\}$.

A) 0
B) 0.1
C) 0.2
D) 0.3
E) 0.4
F) 0.5
G) 0.6
H) 0.7
I) 0.8
J) 0.9

9. Let Σ be the helicoid given by

$$\Sigma = \{ \begin{pmatrix} u\cos v \\ u\sin v \\ v \end{pmatrix} : 0 \leq u \leq 1,\ 0 \leq v \leq \pi \}.$$

Calculate the surface integral $\int\int_{\Sigma} 2y dS$.

A) 1.0
B) 1.1
C) 1.2
D) 1.3
E) 1.4
F) 1.5
G) 1.6
H) 1.7
I) 1.8
J) 1.9

10. A clothesline is hung between two poles 20 feet apart. The line is quite taut and has negligible sag. When a wet shirt weighing 1 pound is hung at the middle of the line, the midpoint is pulled down by 6 inches. Find the magnitude of the tension in the line, in pounds.

A. 10.01
B. 10.02
C. 10.03
D. 10.04
E. 10.05
F. 10.06
G. 10.07
H. 10.08
I. 10.09
J. 10.10

11. Find the distance between the point $(1, 2, 1)$ and the plane $\{2x + 3y - z = 1\}$.

A) 1.60
B) 1.61
C) 1.62
D) 1.63
E) 1.64
F) 1.65
G) 1.66
H) 1.67
I) 1.68
J) 1.69

12. Find the maximum value of $x + 2y + 7z$ on the unit sphere $\{(x, y, z) : x^2 + y^2 + z^2 = 1\}$.

A) 7.0
B) 7.1
C) 7.2
D) 7.3
E) 7.4
F) 7.5
G) 7.6
H) 7.7
I) 7.8
J) 7.9

13. The function $f(x, y) = x^3 - 6x + y^3 + 3y^2 + 2$ has a unique local maximum. Its value at that point is:

A) 11.0
B) 11.1
C) 11.2
D) 11.3
E) 11.4
F) 11.5
G) 11.6
H) 11.7
I) 11.8
J) 11.9

14. Evaluate $\int\int_D y^2 dA$, where D is the upper half of the unit disk, the set $\{(x, y) : x^2 + y^2 \leq 1,\ y \geq 0\}$.

A) 0.30
B) 0.31
C) 0.32
D) 0.33
E) 0.34
F) 0.35
G) 0.36
H) 0.37
I) 0.38
J) 0.39

15. Evaluate $\int\int\int_E x dV$ where E is the tetrahedron bounded by the planes $x = 0,\ y = 0,\ z = 0$ and $x + 2y + z = 2$.

A) 0.1
B) 0.2
C) 0.3
D) 0.4
E) 0.5
F) 0.6
G) 0.7
H) 0.8
I) 0.9
J) 1.0

16. Saying a function $f : \mathbb{R} \to \mathbb{R}$ is continuous at the point a means:

A) For every $\varepsilon > 0$, for every $\delta > 0$, if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
B) There exists $\varepsilon > 0$ such that for every $\delta > 0$, if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
C) For every $\varepsilon > 0$ there exists $\delta > 0$, such that if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
D) There exists $\varepsilon > 0$, there exists $\delta > 0$, such that if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
E) For every $\delta > 0$, for every $\epsilon > 0$, if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
F) There exists $\delta > 0$ such that for every $\epsilon > 0$, if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
G) For every $\delta > 0$ there exists $\epsilon > 0$, such that if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.
H) There exists $\delta > 0$, there exists $\epsilon > 0$, such that if $|x - a| < \delta$ then $|f(x) - f(a)| < \varepsilon$.