Math 233 Exam I September 24, '07

Part I, True or False, 3 points each

Answer A for True, B for False.

1. If $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors then the vector $\mathbf{u} \times \mathbf{v}$ is perpendicular to $\mathbf{u}$.

2. If the vector function $\mathbf{r}(t)$ has constant length, $|\mathbf{r}(t)| = \text{constant}$, then $\mathbf{r}(t)$ is perpendicular to $\frac{d}{dt}\mathbf{r}(t)$.

3. The only curves in three dimensional space with constant curvature (that is, the curvature function $\kappa(\mathbf{u})$ takes the same value at every point of the curve) are circles and straight lines.

4. If two planes are not parallel then they intersect in a line and the direction of that line is parallel to the cross product of the normal vectors for the two planes.

5. If $L(t)$ is the length of the curve $\mathbf{r}(s)$ for $0 \le s \le t$ then $L'(t) = |\mathbf{r}'(t)|$.

Part II, Multiple Choice, 5 points each

1. Find the center and radius of the sphere with equation $x^2 - 2x + y^2 + z^2 + 6z = 10$
   a. Center $(2,3,-1)$, radius $\sqrt{22}$,
   b. Center $(-2,0,-1)$, radius $\sqrt{14}$,
   c. Center $(1,0,-3)$, radius $\sqrt{20}$,
   d. Center $(-2,-3,-1)$, radius $\sqrt{18}$,
   e. Center $(2,0,-1)$, radius $\sqrt{19}$.

2. The vector $\overrightarrow{P_1P_2}$ goes from initial point $P_1 = (2,0,8)$ to terminal point $P_2 = (-1,-2,-3)$. Find the vector of length 1 which points in the same direction as $\overrightarrow{P_1P_2}$.
   a. $\frac{1}{\sqrt{68}}(4,4,6)$,
   b. $\frac{1}{\sqrt{30}}(1,-2,5)$,
   c. $\frac{1}{\sqrt{68}}(4,4,6)$,
   d. $\frac{1}{\sqrt{134}}(-3,-2,-11)$,
   e. $\frac{1}{\sqrt{30/4}}\left(\frac{1}{2},-1,\frac{5}{2}\right)$.

3. Suppose $\mathbf{u} = (2,-2,1)$, $\mathbf{v} = (1,-6,3)$. Compute the length $|3\mathbf{u} - 2\mathbf{v}|$.

a. 7,

b. 8,

c. $\sqrt{23}$,

d. $\sqrt{61}$,

e. $\sqrt{31}$.

4. Suppose $\mathbf{u} = (2,-2,1)$, $\mathbf{v} = (1,-6,3)$, $\mathbf{w} = (2,-2,1)$. Compute $(\mathbf{u} + \mathbf{v}) \times \mathbf{w}$.

a. $(0,5,10)$,

b. $(4,18,-23)$,

c. $(4,1,0)$,

d. $(-6,2,-18)$,

e. $(-12,0,14)$.

5. Find the volume of the (box) parallelepiped determined by the three edges

$$\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$$
$$\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$
$$\mathbf{w} = \mathbf{i} + \mathbf{k}.$$

- **a.** 3,
- **b.** 4,
- **c.** 1,
- **d.** 5,
- **e.** 8.

6. The line L through the point $(3, 1, 7)$ and parallel to the vector $\mathbf{i} + \mathbf{j} - \mathbf{k}$ passes through the point $(1, b, c)$. Find b and c.

- **a.** $b = 1$, $c = -1$,
- **b.** $b = 3$, $c = 5$,
- **c.** $b = -3$, $c = -15$,
- **d.** $b = 3$, $c = -9$,
- **e.** $b = -1$, $c = 9$.

7. Find the cosine of the angle at which the planes $3x - 4y + 2z = 7$ and $2x + 3y + 4z = 8$ intersect.

a. $\frac{8}{\sqrt{17}}$,
b. $\frac{2}{29}$,
c. $\frac{8}{29}$,
d. $\frac{-2}{\sqrt{17}}$,
e. $\frac{1}{\sqrt{17}}$.

8. A particle starts at time $t = 0$ at position $(1,2,3)$ and velocity $\mathbf{v} = (0,0,0)$. It is then subjected to a constant acceleration $\mathbf{a} = (1,0,-2)$. What is the position of the particle at time $t = 1$?

a. $\left(\frac{3}{2},0,-2\right)$,
b. $\left(0,\frac{2}{3},1\right)$,
c. $\left(\frac{3}{2},2,2\right)$,
d. $\left(\frac{1}{2},\frac{3}{2},0\right)$,
e. $(1,1,-2)$.

9. Set $\mathbf{r}(t) = (2t+1)\mathbf{i} + (\sin(t^2))\mathbf{j} - 3\mathbf{k}$, find $\mathbf{r}'(0)$.

- **a.** $\mathbf{r}'(0) = 2\mathbf{i}$,
- **b.** $\mathbf{r}'(0) = 2\mathbf{i} + \mathbf{j}$,
- **c.** $\mathbf{r}'(0) = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$,
- **d.** $\mathbf{r}'(0) = \mathbf{j} - 3\mathbf{k}$,
- **e.** $\mathbf{r}'(0) = -3\mathbf{k}$.

10. Evaluate

$$\int_1^2 \left[\frac{1}{t}\mathbf{i} - 2t\,\mathbf{j} + \mathbf{k} \right] dt.$$

- **a.** $3\,\mathbf{i} - 4\,\mathbf{j} + 2\,\mathbf{k}$,
- **b.** $\ln 2\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$,
- **c.** $\ln 2\,\mathbf{i} + 3\,\mathbf{j} - \mathbf{k}$,
- **d.** $(\arctan 2 - \arctan 1)\,\mathbf{i} - 3\,\mathbf{j} + \mathbf{k}$,
- **e.** $2\,\mathbf{i} - \mathbf{j}$.

11. Find the length of the curve

$$\mathbf{r}(t) = 2t\,\mathbf{i} + 4\cos t\,\mathbf{j} - 4\sin t\,\mathbf{k};\quad 0 \le t \le 2\pi.$$

a. $2\sqrt{2}\,\pi$,
b. $2+\pi$,
c. $2\sqrt{20}\,\pi$,
d. $2+20\pi$,
e. $1+8\pi$.

12. A particle A moves from $(0,0)$ to $(1,1)$ as t varies from 0 to 1. Its position at time t is given by $\mathbf{r}_A(t) = (t, \sqrt{t}\,)$. Particle B moves along from the same start point to the same final point with its position at time t given by $\mathbf{r}_B(t) = (t, t^2)$. For which t between 0 and 1 is the particle A moving faster than particle B?

a. All t,
b. No t,
c. $t > 16^{-1/3}$,
d. $t < 16^{-1/3}$,
e. Can't be determined from the given information.

13. The motion of a particle in space is described by the formula

$$\mathbf{r}(t) = \mathbf{i} + t\mathbf{j} + t^2\mathbf{k}.$$

Find the angle between the velocity vector of the particle and the acceleration vector at time $t = 2$.

a. $\cos^{-1}\left(\frac{-2}{\sqrt{15}}\right) = 2.113,$

b. $\cos^{-1}\left(\frac{12}{\sqrt{65}\sqrt{15}}\right) = 1.176,$

c. $\cos^{-1}\left(\frac{7}{6\sqrt{11}}\right) = 1.211,$

d. $\cos^{-1}\left(\frac{-4}{\sqrt{14}\sqrt{7}}\right) = 1.986,$

e. $\cos^{-1}\left(\frac{8}{2\sqrt{17}}\right) = 0.244.$

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Part III, Hand Graded, 20 points
Show enough work so that it is clear how you arrived at your answer.

The position of a particle at time t is given by

$$\mathbf{r}(t) = (t^2 + 1)\,\mathbf{i} + 2t\,\mathbf{j} - 6\,\mathbf{k}.$$

1. Find the velocity **v**.
2. Find the acceleration **a**.
3. Find the unit tangent vector **T** for the path of the particle.
4. Find the unit normal vector **N** for the path of the particle.
5. Decompose the acceleration into its tangential and normal components. That is, find numbers a_T and a_N so that

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N}.$$

Note: All of these quantities are functions of the variable t.

1. $\mathbf{v} =$.

2. $\mathbf{a} =$

3. $\mathbf{T} =$

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4. $\mathbf{N} =$

5. $a_T =$

$a_N =$