

Part I, True or False, 3 points each. Enter A for True, B for False.

1. If the tangent plane to the graph of $f(x,y)$ at the point $(x_0, y_0, f(x_0, y_0))$ is horizontal then $(\nabla f)(x_0, y_0) = \mathbf{0}$.

2. If $f(x,y)$ is a function defined on a region R then the average of f on the region R is defined to be

$$\text{avg} f = \frac{1}{A} \iint_R f(x,y) dA.$$

3. For a function f of two variables, the directional derivative of f at a point (x,y) in a direction perpendicular to $\nabla f(x,y)$ is zero.

4. The "standard linear approximation" to a function $z = f(x,y)$ near a base point (x_0, y_0) (also called the "approximation using differentials") uses the z coordinate of the tangent plane to the graph of $f(x,y)$ at the base point to approximate the value of the function.

5. The level curves of the function $f(x,y) = (x + y + 4)^2$ are circles centered at the origin.

Part II, Multiple Choice, 5 points each

6. Suppose $f(x,y) = x^2y$ and R is the region between the curves $y = x^2$ and $y = \sqrt{x}$. Evaluate

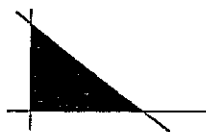
$$\iint_R f(x,y) dA.$$

- a. $2/17$
 - b. $4/17$
 - c. $3/56$
 - d. $14/3$
 - e. $5/14$
7. The function $f(x,y) = x^3 + y^3 - 3xy$ has two critical points, one, A , in the first quadrant (i.e. both coordinates positive) and another, B . Which statement is correct?
- a. A is a saddle point, B is a local minimum.
 - b. A is a local maximum, B is a saddle point.
 - c. A is a local minimum, B is a saddle point.
 - d. A is a saddle point, B is a local maximum.
 - e. A is a local maximum, B is a local minimum.

8. Suppose $w = (x + 2y)^3$ and that $x = r^2 - s$, $y = rs$. Find $\frac{\partial w}{\partial s}$ in terms of r and s .

- a. $3(r^2 - s + 2rs)^2(-1 + 2r)$
- b. $3(r^2 - s + 2rs)(-1 - 2r)$
- c. $3(r^2 - s + 2rs)^2$
- d. $3(r^2 - s + 2rs)^2(-1 + 2r)(2)$
- e. $3(r^2 - s + 2rs)^2(1 - 2r)(2r - s)$

9. Find the maximum and minimum value of the function $f(x, y) = x + 2y$ on the region where $x \geq 0, y \geq 0$, and $x + y \leq 2$. This region is the triangular region, including boundary, with corners at $(0, 0)$, $(2, 0)$, and $(0, 2)$.



- a. No Max. Min = 0,
- b. Max = 3, No Min.
- c. Max = 3, Min = 2.
- d. Max = 4, Min = 0.
- e. Max = 2, Min = 0.

10. Find the directional derivative of the function $f(x,y) = x^2 + 3xy$ at the point $(1,2)$ in the direction of the vector $\mathbf{v} = (1,2)$.
- a. $2/\sqrt{3}$
 - b. 14
 - c. $11/2$
 - d. $14/\sqrt{5}$
 - e. $1/\sqrt{2}$
11. Find the volume of the region bounded above by the surface $z = 3x^2 + 2y^2$ and bounded below by the rectangle $0 \leq x \leq 1, 1 \leq y \leq 2$.
- a. $22/5$
 - b. $17/3$
 - c. 3
 - d. $14/5$
 - e. $7/3$

12. Find the Taylor polynomial of degree two (also called the quadratic approximation) for the function $f(x,y) = \cos x \cos y$ at the base point $(0,0)$.

- a. $1 - xy$
- b. $1 - x^2 - y^2$
- c. $1 - \frac{1}{2}x - \frac{1}{2}y$
- d. $1 - x - y$
- e. $1 - \frac{1}{2}x^2 - \frac{1}{2}y^2$

13. Evaluate the integral

$$\iint_R \frac{1 - \cos x}{x} dA$$

where R is the triangle in the xy -plane bounded by the x -axis, the line $x = y$, and the line $x = 1$.

- a. $(1 - \cos 1)^2/2$
- b. $1 - \cos 1$
- c. $1/4$
- d. $1 - \sin 1$
- e. $(1 - \sin 1)^{-1}$

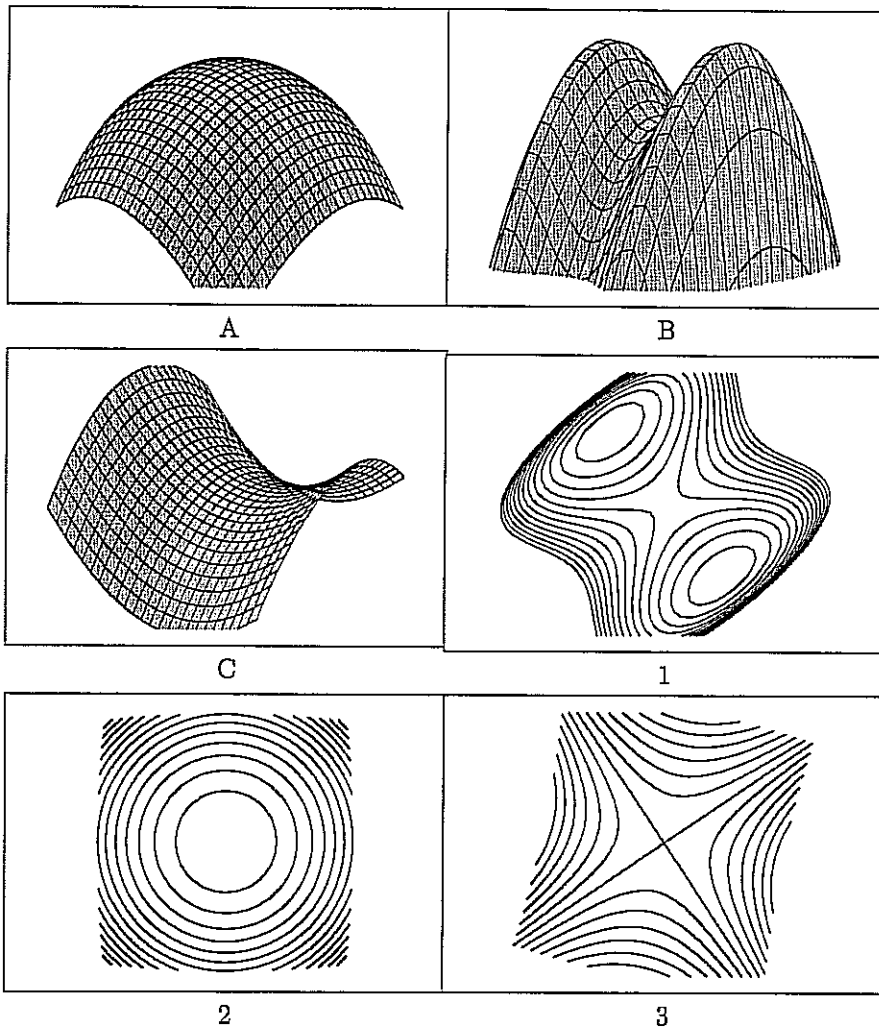
14. Find the equation of the tangent plane to the graph of $f(x,y) = x^2 + y^4$ at the point $(1,1,2)$. Put the equation in the form $x + by + cz = d$.

- a. $x - 3y + \frac{1}{2}z = 2$
- b. $x + 2y - \frac{1}{2}z = 3$
- c. $x + 3y = 2$
- d. $x - 2y + z = 2$
- e. $x - 2y - z = 3$
- f. $x + y + z = 2$

15. The function $f(x,y) = xy - 2x^2 - 2y^2 - 15x + 3$ has one critical point. Find and characterize that critical point.

- a. Saddle point at $(0,0)$
- b. Saddle point at $(-4,-1)$
- c. Local maximum at $(-4,-1)$
- d. Local minimum $(-4,-1)$
- e. Local minimum at $(0,0)$

16. Match the graphs, A, B, C with the corresponding contour line representations, 1, 2, and 3.



- (a) $A \iff 1, B \iff 2, C \iff 3,$
 (b) $A \iff 1, B \iff 3, C \iff 3,$
 (c) $A \iff 2, B \iff 1, C \iff 3,$
 (d) $A \iff 2, B \iff 3, C \iff 1,$
 (e) $A \iff 3, B \iff 2, C \iff 1,$
 (f) $A \iff 3, B \iff 1, C \iff 2,$

17. Find the vector of length one pointing in the direction in which the function $f(x,y,z) = xy + x^2z$ is increasing most rapidly at the point $(1,0,2)$.
- a. $(0,1,0)$
 - b. $(1,0,2)/\sqrt{4}$
 - c. $(0,2,3)/\sqrt{13}$
 - d. $(4,1,1)/\sqrt{18}$
 - e. $(-1,-2,0)/\sqrt{5}$
18. Find the maximum value of the function $f(x,y) = xy^2$ on the curve $x^2 + 2y^2 = 12$.
- a. 32
 - b. 0
 - c. 4
 - d. 18
 - e. 8

Name _____ ID Number _____

Part III, Hand Graded, 20 points. **Show enough work so that it is clear how you arrived at your answer.**

1. Using Lagrange multipliers (or some other method if you wish) find the point on the plane $x + 2y + 3z = 7$ closest to the point $(0, 0, 1)$.

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Part III, Hand Graded, 20 points. Show enough work so that it is clear how you arrived at your answer.

2. Let $f(x, y) = x$ and let R be the triangular region in the xy -plane with vertices $(0, 0)$, $(1, 0)$ and $(1, 1)$. According to Fubini's theorem the integral $\iint_R f(x, y) dA$ can be evaluated two different ways as repeated (iterated) integrals. Set up and evaluate both of these repeated integrals. Be sure to indicate clearly the limits of integration and the integration variables in all integrals.