Math 318 Exam II March 28, '07

Name:____

ID Number:____________________________________

Do your work on this exam booklet. There is one page provided for each problem. Extra paper is available for scratch work. On questions 2, 3, and 4 give enough detail so that it is possible to follow your work.

1	2a	2b	2c	3a	3b	3c	4a	4b

1. (20 points) True or False.

1a. It is impossible to find 4 linearly independent vectors in $\mathbb{R}^3$.

1b. If T is a linear map of $\mathbb{R}^3$ to $\mathbb{R}^3$ which is not the zero map then the function $f(\vec{x}) = \|T\vec{x}\|$ does not have a maximum on the set $\{\vec{x} \in \mathbb{R}^3 : \|\vec{x}\| < 1\}$.

1c. In the vector space of polynomials of degree at most 4 the polynomials $p(x) = x^2$, $q(x) = x$ and $r(x) = 2x^2 - 3x$ are linearly independent.

1d. The set in of all vectors $\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right]$ in $\mathbb{R}^2$ which satisfy the conditions $xy = 0$ and $-1 \leq x - y \leq 1$ is compact.

1e. If $f(\vec{x})$ is a continuous function on the set $S = \{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] : x^2 + y^2 \leq 1 \leq\}$ then f has a maximum value on the set S and that maximum value is taken at a point $\vec{a}$ where $Df(\vec{a}) = \vec{0}$.

Question 1 answers:

a	b	c	d	e

2. (35 points) Do 2 of 3.

2a. Perform a series of elementary operations on the rows of the matrix

$$\begin{pmatrix} 2 & 1 \\ 2 & -1 \\ 4 & 4 \end{pmatrix}$$

to obtain the reduced echelon form of the matrix. What is the rank of the matrix?

2b. For which numbers $a, b,$ and c does the system of equations

$$2x + y = a$$
$$2x - y = b$$
$$4x + 4y = c$$

have a solution.

2c. Suppose $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$ are a basis of $\mathbb{R}^3$. Define $\vec{w}_1, \vec{w}_2$ and $\vec{w}_3$ by

$$\vec{w}_1 = \vec{v}_2 + \vec{v}_3$$
$$\vec{w}_2 = \vec{v}_1 + \vec{v}_3$$
$$\vec{w}_3 = \vec{v}_1 + \vec{v}_2.$$

Must it be true $\vec{w}_1, \vec{w}_2$ and $\vec{w}_3$ are a basis of $\mathbb{R}^3$? Give a complete justification for your answer.

3. (35 points) Do 2 of 3.

3a. Find the maximum and minimum value of the function $f\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = x + 3y$ on the closed box $\left\{\left[\begin{smallmatrix} x \\ y \end{smallmatrix}\right] \in \mathbb{R}^2 : 0 \leq x \leq 1,\ 0 \leq y \leq 1\right\}$.

3b. Find and classify the critical points of the function $f\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = x^3 + y^2 - x - 2xy$.

3c. Use the method of Lagrange multipliers to maximize the function $f\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) = xy^2$ subject to the constraint $x^2 + y^2 = 3$.

4. (10 points) Do 1 of 2.

4a. Suppose $\vec{u}$ and $\vec{v}$ are linearly independent vectors in $\mathbb{R}^4$. For which numbers α, β are the two vectors $\vec{w} = \vec{u} + \vec{v}$ and $\vec{z} = \alpha\vec{u} + \beta\vec{v}$ linearly independent.

4b. Recall that if T is a linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ then the norm of T, $\|T\|$, is defined to be

$$\|T\| = \max_{\|\vec{x}\|=1} \|T\vec{x}\|.$$

Show that is S is another linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ then

$$\|S - T\| \leq \|S\| + \|T\|.$$

1. **2a.** Perform a series of elementary operations on the rows of the matrix

$$\begin{pmatrix} 2 & 1 \\ 2 & -1 \\ 4 & 4 \end{pmatrix}$$

to obtain the reduced echelon form of the matrix. What is the rank of the matrix?

2b. For which numbers $a, b,$ and c does the system of equations

$$\begin{aligned} 2x + y &= a \\ 2x - y &= b \\ 4x + 4y &= c \end{aligned}$$

have a solution.

2c. Suppose $\vec{v}_1, \vec{v}_2$, and $\vec{v}_3$ are a basis of $\mathbb{R}^3$. Define $\vec{w}_1, \vec{w}_2$ and $\vec{w}_3$.

$$\begin{aligned}
\vec{w}_1 &= \vec{v}_2 + \vec{v}_3 \\
\vec{w}_2 &= \vec{v}_1 + \vec{v}_3 \\
\vec{w}_3 &= \vec{v}_1 + \vec{v}_2
\end{aligned}$$

Must it be true $\vec{w}_1, \vec{w}_2$ and $\vec{w}_3$ are a basis of $\mathbb{R}^3$? Give a complete justification for your answer.

3a. Find the maximum and minimum value of the function $f\begin{pmatrix} x \\ y \end{pmatrix} = x + 3y$ in the closed box $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 : 0 \le x \le 1,\ 0 \le y \le 1 \right\}$.

3b. Find and classify the critical points of the function $f\begin{pmatrix} x \\ y \end{pmatrix} = x^3 + y^2 - x - 2xy$.

3c. Use the method of Lagrange multipliers to maximize the function $f\begin{pmatrix} x \\ y \end{pmatrix} = xy^2$ subject to the constraint $x^2 + y^2 = 3$.

4a. Suppose $\vec{u}$ and $\vec{v}$ are linearly independent vectors in $\mathbb{R}^4$. For which numbers α, β are the two vectors $\vec{w} = \vec{u} + \vec{v}$ and $\vec{z} = \alpha\vec{u} + \beta\vec{v}$ linearly independent.

4b. Recall that if T is a linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ then the norm of T, $\|T\|$ is defined to be

$$\|T\| = \max_{\|\vec{x}\|=1} \|T\vec{x}\|.$$

Show that is S is another linear map from $\mathbb{R}^n$ to $\mathbb{R}^m$ then

$$\|S - T\| \leq \|S\| + \|T\|.$$