Math 318 Final Exam May 8, '07

Name:______________________________________

ID Number:__________________________________

Do your work in this exam booklet. There are extra blank pages included at the end. Unless the instructions specify otherwise give enough detail so that it is possible to follow your work.

	1	
Part I	2	
Do 4 of 5	3	
60 points	4	
	5	
Part II	1	
Do 2 of 3	2	
20 points	3	
	1	
Part III	2	
Do 5 of 5	3	
20 points	4	
	5	
Total		

I Do 4 of 5, 15 points each.

1. Find the orthogonal (perpendicular) projection of the vector $\mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ onto the subspace V of $\mathbb{R}^3$ described by the equation $x_1 + x_3 = 0$. Find a vector $\mathbf{y}$ in V so that the vector $\mathbf{x} - \mathbf{y}$ is perpendicular to V.

2. Evaluate the integral $\int_R f\,dA$ for

$$f\begin{pmatrix} x \\ y \end{pmatrix} = x^2 y$$

$$R = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} : x \geq 1/2,\ y \geq 0,\ x^2 + y^2 \leq 1 \right\}.$$

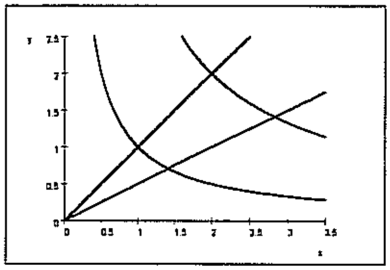
3. Find the volume of the region in $\mathbb{R}^3$ bounded above by the surface $z + x^2 + y^2 = 12$, bounded below by the surface $z = \sqrt{x^2 + y^2}$, and to the right of the plane $x = 0$ (i.e., in the region where $x > 0$).

4. Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 0 \\ 2 & 0 & 0 \end{pmatrix}$. Find $\det A$. Find A^{-1}.

5. Evaluate $\int_R f\, dA$ where

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \frac{x^2 + y^2}{xy}$$

and R is bounded by the four curves $xy = 1$, $xy = 4$, $x = y$, $x = 2y$.



R

II Do 2 of 3, 10 points each.

1. Find the point on the plane $2x + 3y + 4z = 36$ closest to the origin.

2. Use the method of Lagrange multipliers to find the maximum, if there is one, and minimum, if there is one, of the function $f\binom{x}{y} = x + 2y$ subject to the condition that $x^2 + 4y^2 = 16$.

3. What condition, if any, must the numbers a, b, and c satisfy in order for the system of equations

$$\begin{aligned} 2x + 3y + z &= a \\ x + 2y + z &= b \\ 3x + 4y + z &= c \end{aligned}$$

to have a solution? If that condition is satisfied will the solution be unique?

III Do 5 of 5, 4 points each.

1. The matrix A and its inverse A^{-1} are:

$$A = \begin{pmatrix} 1 & 2 & 4 \\ 1 & -2 & 6 \\ -3 & 4 & 5 \end{pmatrix}, \quad A^{-1} = \frac{1}{88}\begin{pmatrix} 34 & -6 & -20 \\ 23 & -17 & 2 \\ 2 & 10 & 4 \end{pmatrix}.$$

Use A^{-1} to solve the system of equations

$$\begin{aligned} x + 2y + 4z &= 6 \\ x - 2y + 6z &= 6 \\ -3x + 4y + 5z &= 6 \end{aligned}$$

2. Suppose A is a linear mapping from $\mathbb{R}^4$ to $\mathbb{R}^5$. The range of A, Range(A), is defined to be all vectors $\mathbf{y}$ in $\mathbb{R}^5$ which can be written as $\mathbf{y} = A\mathbf{x}$ for some $\mathbf{x}$ in $\mathbb{R}^4$; that is, Range(A) $= \{A\mathbf{x} : \mathbf{x} \text{ in } \mathbb{R}^4\}$. Is Range($A$) a subspace of $\mathbb{R}^5$? Justify your answer.

3. Suppose $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ are non-zero vectors in $\mathbb{R}^4$ which are pairwise orthogonal (i.e. $\mathbf{x}\perp\mathbf{y}$, $\mathbf{x}\perp\mathbf{z}$, and $\mathbf{z}\perp\mathbf{y}$). Must it be true that the three vectors are linearly independent? Justify your answer.

4. Give a vector of length 1 pointing in the direction in which the function $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = xy + x^2z - 2yz$ is increasing most rapidly at the point $\mathbf{p} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$. Show your work.

5 Immediately after the definition of derivative the book states, "This says that $D\mathbf{f}(\mathbf{a})$ is the best linear approximation to the function $\mathbf{f} - \mathbf{f}(\mathbf{a})$ at $\mathbf{a}$.". For the given function $\mathbf{f}$ and base point $\mathbf{a}$ give the explicit formula for that "best approximation". Explain briefly and clearly what is "best" about it.

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 + 2y \\ 2xz \\ x - 1 \end{pmatrix}; \quad \mathbf{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$