Math 493, Fall '06, Assignment 4

Due Wednesday, October 4^{th}

Section	Problems
3.2	2, 4, 6, 8
3.3	4 a-f, 6, 8, 10, 12
3.4	4, 6, 8

Exam I, October 4^{th}:

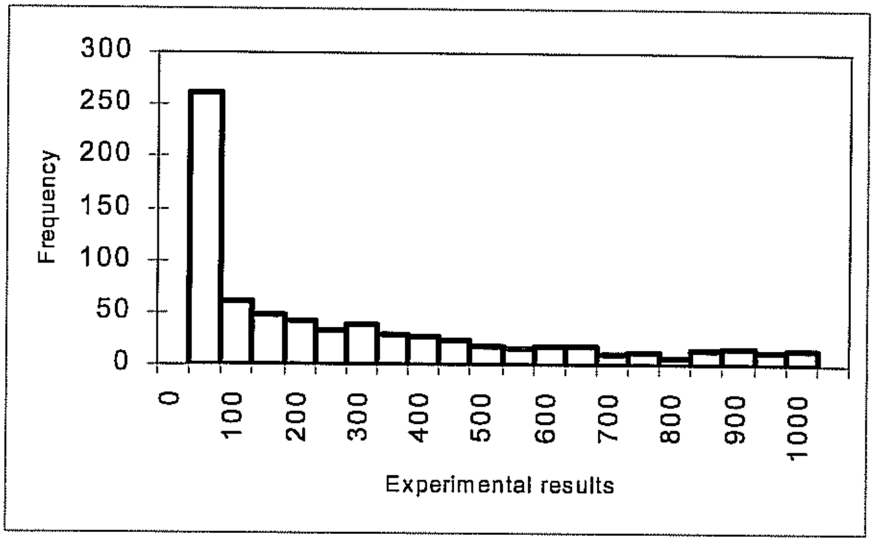
- **In class exam, starting at 2:07.**
- **You may bring a 3 X 5 card with handwritten notes, both sides.**
- **Calculators are allowed but won't be necessary.**
- **Attached is a copy of the first exam from the last time I taught this course. We used a different book and hence there is only a moderate overlap in topics, but you may find it interesting.**

Math 493 Exam I October 5, '01

You don't need to reduce your answers to decimal form.

Part 1: Do all 3; 10 points each

1. The results of 723 repetitions of an experiment which produces outcomes between 50 and 1050 are summarized in the histogram. The first bar represents the bin (50,100), the last (1000,1050). Give a rough estimate of the mean and quartiles of the data.

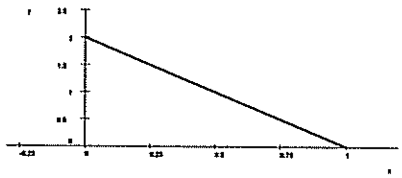


2. A,B,C, and D are events; C and D are independent; and $P(D) \neq 0$. A' is the complement of A, etc. Is it always True or sometimes False that: (no reasons need be given)
   - a. $P(A \cup B) = P(A) + P(B)$
   - b. $P(A) + P(A') = 1$
   - c. $P(C \cup D) = P(C) + P(D) - P(C)P(D)$
   - d. $P(C' \cap D) = P(C')P(D)$
   - e. $P(C|D) = P(C)$
3. Suppose $P(A \cap B') = .1$, $P(A \cap B) = .3$, $P(A' \cap B) = .4$
   - a. What is $P(A)$?
   - b. What is $P(A \cup B)$?

Part 2: Do 3 Of 5; 10 points each

1. An urn contains 3 red balls and 6 yellow. What is the probability that if you pick three balls at random they will be the same color if
   - a. the selection is done with replacement?
   - b. the selection is done without replacement?
2. Suppose that your lucky day happens once a year. On your lucky day there is a 70% chance that something really good will happen to you. On other days there is only a 1% chance that something really good will happen to you. Suppose that something really good happens to you today. What is the probability that today is your lucky day.
3. An experiment produces outcomes **X** with probabilities given by the probability density function

$$f(x) = \begin{cases} 2-2x, & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

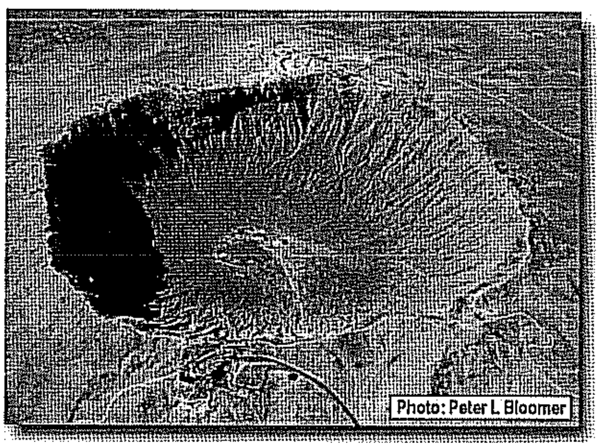


a. What is the mean of this random variable?
b. What is the median?
c. What is the expectation of the function $U(\mathbf{X}) = \mathbf{X}^2$?

4. The random variable X is uniformly distributed on the interval $(1.5, 5.5)$. What is the probability that the digit of X to the right of the decimal point is a 3?
5. The moment generating function of the random variable X is given by $M(t) = \frac{1}{2}e^{-t} + \frac{1}{2}e^{2t}$.
   a. Find the standard deviation of X.
   b. Find $P(X > 0)$.

Part 3: Do 1 of 3; 20 points each

1. A meteor about 75 yards wide made a crater almost a mile wide in Arizona.



The most recent impact of this magnitude was in the Tunguska region of Siberia in 1908. Scientists estimate that such events occur on average once every 1000 years.
   a. What would have been your estimate in 1908 of another such event within 50 years.
   b. Actually there have been no such events since then. What would be your current estimate of the probability that such an event will happen within the next 50 years (that is, roughly, in your lifetime).
   c. What is the mean time we would expect to wait for the next such event?
   d. What is the median time?
2. A manufacturing process produces items 3% of which are defective.
   a. What is the probability that there are two or more defective items in a batch of 100.
   b. Use an appropriate Poisson approximation to solve (approximately) the previous problem.
   c. Suppose that the batch of 100 items actually contains 5 defective items. What is the

probability that if you selected 10 from that batch you would find 2 or more defectives?

3. You roll a fair 6 sided die k times.
   a. $k > 4$; what is the probability that you obtain your second 5 on the 4th roll?
   b. $k > 4$; what is the probability that the 2nd and 4th rolls produce the same results?
   c. $k = 10$; what is the probability of obtaining exactly three 2's in these 10 rolls?
   d. $k = 3$; compute the probability that you get 3 different outcomes.
   e. $k = 3$; compute the conditional probability that the product of the faces you see is even given that you observed 3 different outcomes.