Math 493 Fall '06 Exam I Solutions

1) a) 36 possible pairs, equally likely
sum > 9 : (6,4) (5,5) (4,6) (5,6) (6,5) (6,6)
Ans = 6/36 = 1/6

b) Two dice (E,E) (E,O), (O,E), (O,O)
equally likely, 3 of 4 have even product
Ans = 3/4

2 Total # deals $N = \binom{52}{13\ 13\ 13\ 13}$

a) # Hands with 4 aces for A: $A = \binom{4}{1\ 1\ 1\ 1}\binom{48}{9\ 13\ 13\ 13}$
ans = A/N = .00264

b) A gets 4, ..., D gets 4 disjoint so
add probs; Ans = 4A/N = .01056

c) B = # ways = $\binom{4}{1\ 1\ 1\ 1}$ $\binom{48}{12\ 12\ 12\ 12}$
distribute aces — distribute rest

Ans = B/N = .10551

D) C = # ways = $\binom{4}{2}$ $\binom{4}{2}$ $\binom{48}{11\ 11\ 13\ 13}$
pick which two — distribute Aces — fill rest of Hand

Ans = C/N = .0337

(2)

3 a) $\int_0^3 cx^2\,dx = 1 \rightarrow c\frac{x^3}{3}\Big|_0^3 = 1 \rightarrow 9c = 1$

$c = 1/9$

b) $Pr(x>2 / x>1) = \dfrac{Pr(x>2 \text{ and } x>1)}{Pr(x>1)}$

$= \dfrac{Pr(x>2)}{Pr(x>1)} = \dfrac{\int_2^3 \frac{1}{9}x^2\,dx}{\int_1^3 \frac{1}{9}x^2\,dx}$

$= \dfrac{\frac{1}{9}\frac{x^3}{3}\Big|_2^3}{\frac{1}{9}\frac{x^3}{3}\Big|_1^3} = \dfrac{27-8}{27-1} = \dfrac{19}{26}$

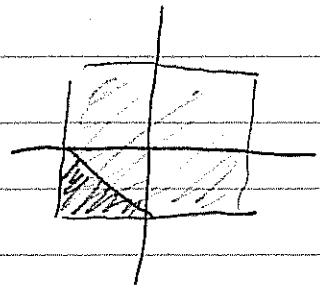
c) Median $= m$

$\frac{1}{2} = \int_0^m \frac{1}{9}x^2\,dx = \frac{1}{27}m^3$

$m = \sqrt[3]{27/2} = .238$

d) $F(1) = Pr(x \le 1) = \int_0^1 \frac{1}{9}x^2\,dx$

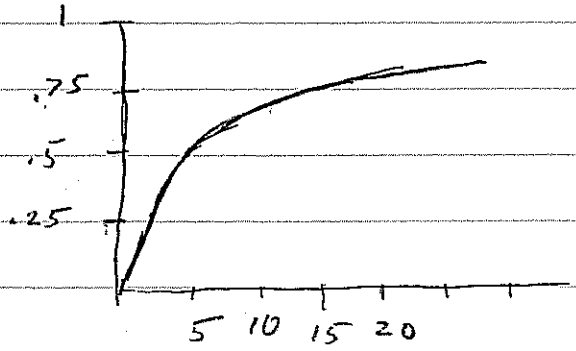
$= \frac{x^3}{27}\Big|_0^1 = \frac{1}{27}$

4) 

uniform distribution on R

$Pr = \frac{\text{Area of interest}}{\text{Area of R}}$

$= \frac{1/2}{4} = \frac{1}{8}$



a) $Pr(X < -1) = 0$, b) $Pr(x < 3) = .3$

c) $Pr(2 < x < 8) = F(8) - F(2) = .65 - .1 = .55$

d) $Pr(X = 8) = 0$ e) median = that x with $F(x) = .5$

median = 5

(4)

5) a) $Pr(A*A) = Pr(AAA) + Pr(ABA) + Pr(ACA)$

$A \to A \to A$ $\quad$ $A \to B \to A$ $\quad$ $A \to C \to A$

$.2 \times .2$ $\quad$ $.4 \times .4$ $\quad$ $.4 \times .4$

$.04 + .16 + .16$

$= .36$

b) AAC or ABC or ACC

$.2 \times .4 + .4 \times .4 + .4 \times .2$

$.08 + .16 + .08$

$= .32$

c) Same as a) because of Markov property.

(5)

6. W = Have white, R = " , B =

$$P(RWB) = 1 - P(A^c \cup R^c \cup B^c)$$
$$= 1 - (P(A^c) + P(R^c) + P(B^c) - P(A^c R^c) - P(A^c B^c) - P(R^c B^c) + P(A^c R^c B^c))$$
$$= 1 - 3P(A^c) + 3P(A^c B^c)$$
$$= 1 - 3\left(\frac{\binom{20}{3}}{\binom{30}{3}}\right) + 3\left(\frac{\binom{10}{3}}{\binom{30}{3}}\right)$$

7. a) $Pr(2) = Pr(2|A)\,Pr(A) + Pr(2|B)\,Pr(B)$
$$= \frac{1}{6}\cdot\frac{1}{2} + \frac{3}{6}\cdot\frac{1}{2} = 3/12 = 1/4$$

b) $$Pr(A|22) = \frac{Pr(22|A)\,Pr(A)}{Pr(22|A)\,Pr(A) + Pr(22|B)\,Pr(B)}$$
$$= \frac{\frac{1}{36}\cdot\frac{1}{2}}{\frac{1}{36}\cdot\frac{1}{2} + \frac{4}{36}\cdot\frac{1}{2}} = \frac{1}{1+4}$$
$$= .2$$