Math 493 Exam II November 1, '06

You don't need to reduce your answers to decimal form.

Part 1: True or False, no reasons need be given in this section, 4 points each. (T = always true; F = not always true.)

1. If $E(X)$ is the expectation of the continuous random variable X then $\Pr(X < E(X)) = .5$.
2. If the joint distribution of the pair of random variables X and Y is uniform over the square with corners $(0,0)$, $(1,0)$, $(0,1)$, and $(1,1)$; then X and Y are independent.
3. If the continuous random variables X and Y are independent then the density function for the conditional density of X given that $Y = y$, $g_1(x\,|\,y)$, is the same as $f_1(x)$, the marginal density of X.
4. If the random variable X is uniformly distributed on the interval $[2,3]$ then the random variable X^2 is uniformly distributed on the interval $[4,9]$.
5. If X is any continuous random variable and $F(x)$ is its distribution function then the random variable $Y = F(X)$ is uniformly distributed on the interval $[0,1]$.

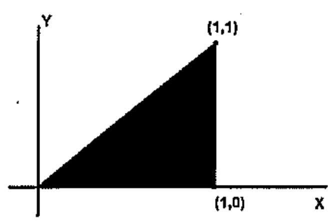
Part 2: Do 4 of 7, 20 points each.

Show enough work or explanation so that it is clear how you arrived at your answer.

1. Suppose the random variables X_1, X_2 and X_3 are independent and each uniformly distributed on the interval $[1,3]$. Find:
   a. the probability $\Pr(X_1 < 2X_2)$,
   b. the expected value $E(X_1^2 - 2X_1X_2)$, and
   c. the probability that $W = \text{Max}(X_1, X_2, X_3) > 2.5$.

2. The density function $f(x,y)$ of the random variables X, Y is

$$f(x) = \begin{cases} 2x + 2y & \text{if } (x,y) \text{ is in the shaded triangle} \\ 0 & \text{otherwise.} \end{cases}$$



   Find
   a. $\Pr(X < 1/2)$,
   b. $\Pr(X = 1/4)$,
   c. the expected value $E(Y)$.

3. The joint p.f. of the discrete random variables X and Y are given in the following table:

		Y = 1	2	3
X	1	.1	.05	.1
	2	.3	.2	.25

a. Are X and Y independent?
b. Find the marginal probability function of Y.
c. Find the conditional probability $\Pr(Y = 2 \mid X = 2)$.
d. Find the probability $\Pr(X^2 + Y^2 > 5)$.

4. The pair of random variables X_1 and X_2 are uniformly distributed over the interior of the triangle with vertices $(x_1,x_2) = (0,0),\ (1,0),\ (1,2)$. The variables Y_1 and Y_2 are functions of X_1, X_2

$$Y_1 = r_1(X_1,X_2) = 2X_1 + X_2,$$
$$Y_2 = r_2(X_1,X_2) = 2X_1 - X_2.$$

a. Find the transformations s_1 and s_2 so that $s_1(Y_1,Y_2) = X_1, s_2(Y_1,Y_2) = X_2$.
b. Where is the density function for Y_1 and Y_2 not equal to zero.
c. On that region what is the formula for the density function.

5. Suppose the random variable X is uniformly distributed on the interval $[-2,2]$. Find
a. the density function of the random variable $V = X^3$, and
b. the density function of the random variable $W = X^4$.
c. Suppose Y has the same distribution as X and is independent of X. Find the probability $\Pr(X^2 + Y^2 \leq 3)$.

6. The pair of random variables X and Y is uniformly distributed on the disk centered at the origin and of radius one; $\{(x,y) : x^2 + y^2 \leq 1\}$. Find the density function of the random variable R = distance of (X, Y) to the origin $(0,0)$; $R = \sqrt{X^2 + Y^2}$.

7. The random variables X_1 and X_2 are independent and identically distributed with density functions

$$f(x) = \begin{cases} 2e^{-2x} & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}.$$

a. Find the joint density function $g(x_1,x_2)$ of the pair X_1, X_2.
b. Find the density function $h(z)$ of the random variable $Z = X_1 + X_2$.

(102) binomial

$$f(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & \text{if } x = 0, 1, ..., n \\ 0 & \text{otherwise} \end{cases}$$

(129) marginal

$$f_1(x) = \int_{-\infty}^{\infty} f(x, y) dy$$
$$f_2(y) = \int_{-\infty}^{\infty} f(x, y) dx$$

(139) conditional

$$g_1(x \mid y) = \frac{f(x, y)}{f_2(y)}$$
$$g_2(y \mid x) = \frac{f(x, y)}{f_1(x)}$$

(161) $Y = r(X)$: if $r(x)$...then for y in the range ...

$$g(y) = f(s(y)) \left| \frac{d}{dy} s(y) \right|$$

(166) $Y_n = \max(X_1, ... X_n)$, $Y_1 = \min(X_1, ..., X_n)$

$$g_n(y) = n \left[F(y) \right]^{n-1} f(y)$$
$$g_1(y) = n \left[1 - F(y) \right]^{n-1} f(y)$$
$$g(y_1, y_n) = n(n-1) \left[F(y_n) - F(y_1) \right] f(y_1) f(y_n)$$

(168) $Y_1 = r_1(X_1, X_2)$, $Y_2 = r_2(X_1, X_2)$

$$g(y_1, y_2) = f(s_1(y_1, y_2), s_2(y_1, y_2)) \left| \frac{\partial s_1}{\partial y_1} \frac{\partial s_2}{\partial y_2} - \frac{\partial s_1}{\partial y_2} \frac{\partial s_2}{\partial y_1} \right|$$

(172) convolution

$$g(y) = \int_{-\infty}^{\infty} f_1(y - z) f_2(z) dz$$

(185, 190) expectation

$$E(r(X)) = \int_{-\infty}^{\infty} r(x) f(x) dx$$
$$E(aX + b) = aE(X) + b$$
$$E(XY) = E(X)E(Y) \ \boxed{\text{IF...}}$$

Math 493, Fall '06, Assignment 9

Due Wednesday, November 8th

Section	Problems
4.3	2, 4, 6,
4.4	2, 4, 6, 8, 10, 12