Math 494 Exam II March 23, '09

- This is a take home exam. It is due at the beginning of class Wednesday.
- You may use your textbook, your class notes, and class handouts. Except for computational web applets you may not use other texts or web resources.
- You may use whatever computational assistance you care to, calculator, computer or web applet. Give enough description of the processes you use so that it is clear that you know what information you have gotten from the software. For instance if you use software that gives output such as "$F-$value = 3.54" you should state which parameters m and n the F is associated with. You should also explain which quantile the offered value represents; that is, it will be the $1-\alpha$ quantile; you should indicate the value of α.
- For hypothesis tests state clearly the null hypothesis and, if appropriate, the alternative. For hypothesis testing or confidence intervals you may select the parameters, α, β etc. Say which values you use and if you use unusual values explain why.
- In general, present enough information so that it I can follow your approach.

Do all problems
Return both a filled in answer sheet (last page) and a BlueBook with your work.
Put your name on BOTH.

1. (a) Suppose a 6-sided die is thrown 60 times and the frequency with which the faces show up is

Face	1	2	3	4	5	6
Frequency	12	14	8	8	8	10

Test the hypothesis that the die is fair, that all faces are equally likely.

(b) Test the hypothesis that the data

Value	0	1	2
Frequency	10	20	30

represents observations from a binomial random variable with $n = 2$ and p unspecified.

2. Suppose a simple of size 3 from a normal random variable of unknown mean m and variance σ^2 gives the values $0, 1, 2$.

   (a) Find a 90% confidence interval for μ.

   (b) Find a 90% confidence interval for σ^2.

   (c) Suppose you use this data to test the null hypothesis $H_0 : \mu = 1$ against the alternative $H_1 : \mu = 2$ with $\alpha = .05$. What is the value of β, the probability of Type II error?

3. Assume the hypothesis of the linear model are met. For the given set of (x, y) values:

x	1	2	3	3
y	7	2	4	2

   (a) Compute the regression line $\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$.

   (b) Compute a 90% confidence interval for the unknown variance σ^2.

   (c) Test the hypothesis $H_0 : \beta_1 = 0$ against the one sided alternative $H_1 : \beta_1 > 0$.

   (d) Give the 95% prediction interval for y corresponding to $x = 1.5$.

4. From Wikipedia, the free encyclopedia

   William Sealy Gosset (June 13, 1876–October 16, 1937) is famous as a statistician, best known by his pen name Student and for his work on Student's t-distribution.

   Born in Canterbury, England to Agnes Sealy Vidal and Colonel Frederic Gosset, Gosset attended Winchester College before reading chemistry and mathematics at New College, Oxford. On graduating in 1899, he joined the Dublin brewery of Arthur Guinness & Son.

   Guinness was a progressive agro-chemical business and Gosset would apply his statistical knowledge both in the brewery and on the farm—to the selection of the best yielding varieties of barley. Gosset acquired that knowledge by study, trial and error and by spending two terms in 1906–7 in the biometric laboratory of Karl Pearson. Gosset and Pearson had a good relationship and Pearson helped Gosset with the mathematics of his papers. Pearson helped with the 1908 papers but he had little appreciation of their importance. The papers addressed the brewer's concern with small samples, while the biometrician typically had hundreds of observations and saw no urgency in developing small-sample methods.

   Another researcher at Guinness had previously published a paper containing trade secrets of the Guinness brewery. To prevent further disclosure of confidential information, Guinness prohibited its employees from publishing any papers regardless of the contained information. This meant that Gosset was unable to publish his works under his own name. He therefore used the pseudonym Student for his publications to avoid their detection by his employer. Thus his most famous achievement is now referred to as Student's t-distribution, which might otherwise have been Gosset's t-distribution.

   Gosset had almost all of his papers including The probable error of a mean published in Pearson's journal Biometrika using the pseudonym Student. However, it was R. A. Fisher who appreciated the importance of Gosset's small-sample work, after Gosset had written to him to say I am sending you a copy of Student's Tables as you are the only man that's ever likely to use them!. Fisher believed that Gosset had effected a "logical revolution".

Gosset's most fundamental paper on this was "The probable error of a mean". Biometrika 6 (1): 1–25. March 1908. In that paper we find

Section IX: Illustration of Method

Illustration I. As an instance of the kind of use which may be made of the tables, I take the following figures from a table by A. R. Cushny and A. R. Peebles in the Journal of Physiology for 1904, showing the different effects of the optical isomers of hyoscyamine hydrobromide in producing sleep. The average number of hours' sleep gained by the use of the drug is tabulated below.

The conclusion arrived at was that in the usual doses 2 was, but 1 was not, of value as a soporific.

Patient	1(Dextro-)	2 (Laevo-)	Difference (2 − 1)
1	.7	1.9	1.2
2	−1.6	.8	2.4
3	−.2	1.1	1.3
4	−1.2	.1	1.3
5	−.1	−.1	0
6	3.4	4.4	1.0
7	3.7	5.5	1.8
8	.8	1.6	.8
9	0	4.6	4.6
10	2	3.4	1.4
mean	.75	2.33	1.58
$\hat{\sigma}$	1.70	1.90	1.17

Additional hours'sleepgained by the use of hyoscyamine hydrobromide

Denote by μ_1 and μ_2 the means of the random variables which underlie the data in the first two columns. Assume those two random variables are normal and have the same variance. Find the P value associated with testing

(a) $H_0 : \mu_1 = 0$ vs. $H_1 : \mu_1 > 0$

(b) $H_0 : \mu_2 = 0$ vs. $H_1 : \mu_2 > 0$

(c) $H_0 : \mu_2 = \mu_1$ vs. $H_1 : \mu_2 > \mu_1$

Answer Sheet

Name: ID Number:

Answers:

1. A)

   .

   B)

   .

2. A)

   .

   B)

   .

   C)

   .

3. A)

   .

   B)

   .

   C)

   .

   D)

   .

4. A)

   .

   B)

   .

   C)

.

Grade:	1		2			3				4			Total
	A	B	A	B	C	A	B	C	D	A	B	C	