Math 494 Exam I February 13, '09

1. A geometric random variable $\mathbf{X}$ is one of a family of random variables depending on a parameter p, $0 < p < 1$. It is defined by

$$P(\mathbf{X} = k) = \begin{cases} (1-p)p^k \text{ for } k = 0, 1, 2, ... \\ 0 \quad \text{otherwise} \end{cases}$$

It has mean $p/(1-p)$ and variance $p/(1-p)^2$. Suppose you have a sample of size n with values $k_1, ..., k_n$.

   (a) Find the formula for the method of moments estimate of p based on that sample.

   (b) Find the formula for the maximum likelihood estimate of the same parameter using the same data.

2. Suppose $\mathbf{X}$ is a normal random variable with unknown mean μ and with standard deviation $\sigma = 4$. A random sample of size 2 from $\mathbf{X}$ produces the values $x_1 = 2$, $x_2 = 4$.

   (a) What is the numerical value of the method of moments estimate of μ?

   (b) What is the numerical value of the maximum likelihood estimate of μ?

   (c) Construct a 95% confidence interval for μ.

   (d) How large a sample would you need to be able to create a 95% confidence interval for μ of total length .1?

3. Suppose $\mathbf{X}$ is a normal random variable with unknown mean μ and standard deviation $\sigma = 5$. Construct a hypothesis test of the null hypothesis $H_0 : \mu = 2$ against the alternative hypothesis $H_1 : \mu \neq 2$ based on a sample of size $n = 25$ and having a significance level $\alpha = .01$.

4. Suppose that you have a sample of size $n = 100$ from a Bernoulli random variable $\mathbf{X}$;

$$P(\mathbf{X} = k) = \begin{cases} p & \text{for } \ k = 1 \\ 1-p & \text{for } k = 0 \end{cases}.$$

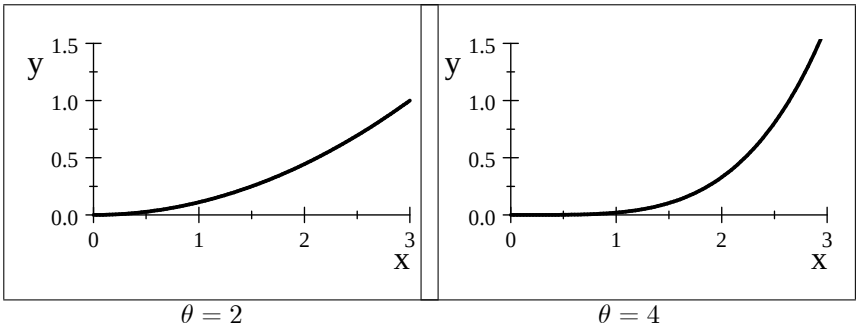
   (a) Describe a hypothesis test of the null hypothesis $H_0 : p = .6$ against the alternative hypothesis $H_1 : p \neq .6$ with $\alpha = .05$.

   (b) Suppose that 65 times your sample has $\mathbf{X} = 1$ and 35 times and $\mathbf{X} = 0$ 35. What would be a 95% confidence interval for estimating p?

5. Consider the family of random variables $\mathbf{X}$ which depend on a parameter θ and are described by the density function

$$\begin{aligned} f_{\mathbf{X}}(x) &= \frac{\theta+1}{3^{\theta+1}}x^{\theta} \quad \text{if } 0 < x < 3 \\ &= 0 \qquad\qquad \text{otherwise.} \end{aligned}$$

The graphs for two values of θ are shown.



$\theta = 2$ $\qquad\qquad$ $\theta = 4$

Suppose you wish to test the null hypothesis $H_0 : \theta = 2$ against the alternative $H_1 : \theta = 4$ using a sample of size 1 and having $\alpha = .05$.

(a) Describe a reasonable test.

(b) What is the value of β for that test?

Table of the Standard Normal (z) Distribution

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00	0.00	0.01	0.01	0.02	0.02	0.02	0.03	0.03	0.04
0.1	0.04	0.04	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08
0.2	0.08	0.08	0.09	0.09	0.09	0.10	0.10	0.11	0.11	0.11
0.3	0.12	0.12	0.13	0.13	0.13	0.14	0.14	0.14	0.15	0.15
0.4	0.16	0.16	0.16	0.17	0.17	0.17	0.18	0.18	0.18	0.19
0.5	0.19	0.20	0.20	0.20	0.21	0.21	0.21	0.22	0.22	0.22
0.6	0.23	0.23	0.23	0.24	0.24	0.24	0.25	0.25	0.25	0.25
0.7	0.26	0.26	0.26	0.27	0.27	0.27	0.28	0.28	0.28	0.29
0.8	0.29	0.29	0.29	0.30	0.30	0.30	0.31	0.31	0.31	0.31
0.9	0.32	0.32	0.32	0.32	0.33	0.33	0.33	0.33	0.34	0.34
1.0	0.34	0.34	0.35	0.35	0.35	0.35	0.36	0.36	0.36	0.36
1.1	0.36	0.37	0.37	0.37	0.37	0.37	0.38	0.38	0.38	0.38
1.2	0.38	0.39	0.39	0.39	0.39	0.39	0.40	0.40	0.40	0.40
1.3	0.40	0.40	0.41	0.41	0.41	0.41	0.41	0.41	0.42	0.42
1.4	0.42	0.42	0.42	0.42	0.43	0.43	0.43	0.43	0.43	0.43
1.5	0.43	0.43	0.44	0.44	0.44	0.44	0.44	0.44	0.44	0.44
1.6	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45	0.45
1.7	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46	0.46
1.8	0.46	0.46	0.47	0.47	0.47	0.47	0.47	0.47	0.47	0.47
1.9	0.47	0.47	0.47	0.47	0.47	0.47	0.48	0.48	0.48	0.48
2.0	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.48
2.1	0.48	0.48	0.48	0.48	0.48	0.48	0.48	0.49	0.49	0.49
2.2	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49
2.3	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49
2.4	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49
2.5	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.49	0.50	0.50
2.6	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.7	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.8	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
2.9	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
3.0	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50