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Inference About the Ratio of Two Parameters, With Application to Whale Censusing

Adrian E. RAFTERY and Tore SCHWEDER*

Inference for the quotient of two parameters estimated separately may be obtained by the delta method. When the distribution of linear transformations involving the numerator and the denominator is available, more exact and elementary methods may be used. Non-Bayesian and Bayesian approaches are developed. The application of the methods to estimating the stock abundance of Northeastern Atlantic minke whales, where the ratio is a raw estimate divided by a measure of observation efficiency, is explained and discussed. The Bayesian approach allows exact inference in quite general situations using only a single, rapidly implemented, one-dimensional numerical integration. A simple analytic approximation is given for the common situation where the joint posterior distribution of the numerator and denominator can be approximated by a normal distribution that gives very little probability to negative values of the denominator. The Bayesian approach also permits the incorporation of model uncertainty (or disagreement) in a natural way, and this was the basis for the conclusions of the International Whaling Commission Scientific Committee at its 1990 meeting.

KEY WORDS: Abundance estimation; Bayesian statistics; Line transect survey; Numerical integration; Population size estimation.

1. INTRODUCTION

Suppose inference is to be made about a parameter θ , which we can write as $\theta = \psi_1/\psi_2$, where inference about ψ_1 and ψ_2 can be made separately. This will be the case, for example, if ψ_1 and ψ_2 are estimated from different experiments. This arises in population size estimation, where θ is the population size and ψ_2 is a detection probability. We encountered it in the context of estimating the abundance of minke whales in the Northeast Atlantic (Schweder, Øien, and Høst 1991).

Inference about a ratio of parameters appears to have been considered first by Bliss (1935a, b) in the context of bioassay. Fieller (1940) presented a solution based on a pivotal quantity and he argued that this was a fiducial solution (Fieller 1944, 1954). Creasy (1954) presented a different solution which she claimed was a fiducial one, but Barnard (1954) and Fisher (1954) dis-

agreed and argued that Fieller's was the true fiducial distribution. The Fieller solution also provides standard confidence intervals, and in Section 3 we give a simple derivation of his method from this point of view.

In Section 4 we give the general Bayesian solution to the problem of inference about θ based on the joint posterior distribution of ψ_1 and ψ_2 . We also give a simple analytic approximation for situations such as that of the minke whale census, where the joint posterior distribution of ψ_1 and ψ_2 can be approximated by a bivariate normal distribution that assigns negligible probability to negative values of ψ_2 . Kappenman, Geisser, and Antle (1970) gave an exact Bayesian solution in the special case of inference about the ratio of the two means given a sample from a bivariate normal distribution, while Zellner (1965, 1978) and Press (1969) gave a Bayesian solution to the problem of inference about a ratio of two regression parameters when the errors are normally distributed. We also describe how this framework can be used to take account of model uncertainty, or, as in the minke whale case, disagreement about model assumptions.

Quotients of parameters are of interest in other situations. Seber (1982) considered a quotient of regression parameters using a method based on that of Fieller (1940). This method was used by Tillman and Breiwick (1983) when estimating the abundance of sperm whales from historical data. Other cases of interest include odds ratios in binomial experiments.

In Section 2 we present the essentials of the minke whale abundance problem and in Section 5 the application of the methods to the minke whale problem is described.

2. THE PROBLEM OF ESTIMATING MINKE WHALE ABUNDANCE FROM SHIPBOARD SURVEYS

Shipboard surveys have been used for more than a decade to estimate the abundance of Southern Hemisphere minke whales (Hiby and Hammond 1989) and in recent years this method has also been used for North Atlantic minke whales. We will refer to the Norwegian survey in 1989 for the Eastern part of the North Atlantic (Øien 1991). Shipboard surveys are special cases of line transect experiments (Burnham, Anderson, and Laake 1980). The survey vessel moves at a constant speed of 10 nautical miles per hour, along randomly chosen track lines. There are two observers in the crow's nest who constantly scan the surface of the ocean in search of a surfacing whale. When a whale is spotted, the radial distance from the observer to the surfacing is measured by eye and the angle between the track line and the sighting line is read off from an angular board.

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From the observed radial distances and angles of the initial observations of a whale, the effective search width is estimated by fitting a curve to the observed distribution of perpendicular distances from the track line to the surfacings. A favorite method for this curve fitting is that of Hayes and Buckland (1983). Øien (1991) found by this method that the unadjusted estimate (see below) of the number of minke whales feeding in the Northeast Atlantic was $\hat{\psi}_1 = 34,800$, with a standard error of $s_1 = 5,500$. The survey covered a total of 13,858 nautical miles steamed by nine vessels during the month of July.

In recent years it has been realized that this method has to be adjusted. If unadjusted it yields stock abundance estimates that are biased downwards because it is based on the assumption that all whales on the track-line are sighted. The northern minke whale is, however, hard to spot. It breaks the surface smoothly for breathing, without releasing a visible blow. The surfacing takes two to three seconds and the number of surfacings at sighting distance is seldom more than seven, and may be zero. The ship moves fast (309 m/minute) and the whale takes dives sometimes lasting more than 10 minutes; the average surfacing rate is around 46.4 surfacings per hour. The probability ψ_2 of sighting a whale that stays on the track line is therefore substantially less than one. With θ being the number of whales, we have $\psi_1 = \theta\psi_2$, and the problem is then to make inference about the ratio $\theta = \psi_1/\psi_2$.

To estimate ψ_2 , an experiment involving two survey vessels on a parallel course was conducted off Spitsbergen in 1989. The method was to estimate the hazard probability of sighting from the set of sightings made by each vessel separately and jointly. A spatial complementary log-log model for the hazard probability was estimated from a sample of 496 binary observations. The hazard probability was then product integrated with respect to the observed surfacing processes of northern minke whales. The result was $\hat{\psi}_2 = .427$ with a standard error of $s_2 = .0542$ (Schweder et al. 1991).

The estimation of stock abundance for whales is of primary interest to the International Whaling Commission (IWC). The research referred to above was extensively discussed at the IWC Scientific Committee meeting in 1990. In Section 5 of this article, we give the results of the methods described in Sections 3 and 4, and we describe some of the ensuing deliberations of the IWC Scientific Committee.

3. NON-BAYESIAN HYPOTHESIS TESTING FOR QUOTIENT PARAMETERS AND RELATED CONFIDENCE INTERVALS

Suppose that the problem is to test $H_0: \psi_1/\psi_2 = \theta_0$ against $H_1: \psi_1/\psi_2 \neq \theta_0$, or equivalently, $H_0: \psi_1 - \theta_0\psi_2 = 0$ against $H_1: \psi_1 - \theta_0\psi_2 \neq 0$.

In the application we have naturally that $\psi_1 \geq 0$ and $\psi_2 > 0$, and we assume this throughout. Let $(\hat{\psi}_1, \hat{\psi}_2)$ be unbiased and jointly normally distributed. Assume furthermore that estimates of standard deviations and correlations are available, which allow for an estimate s_0^2

of the variance of $\psi_1 - \theta_0\psi_2$ to have a distribution equal to that of a constant times a χ_r^2 random variable and to be independent of $\hat{\psi}_1 - \theta_0\hat{\psi}_2$. The two-sided t test based on

$$T = \frac{\hat{\psi}_1 - \theta_0\hat{\psi}_2}{s_0}$$

with ν degrees of freedom is then the optimal test to use, being uniformly most powerful among unbiased tests under appropriate conditions.

When s_1 and s_2 are the standard errors of $\hat{\psi}_1$ and $\hat{\psi}_2$ and r is the estimated correlation such that

$$s_0^2 = s_1^2 - 2\theta_0rs_1s_2 + \theta_0^2s_2^2,$$

the rejection region for the t test is

$$|\hat{\psi}_1 - \theta_0\hat{\psi}_2| > t(s_1^2 - 2\theta_0rs_1s_2 + \theta_0^2s_2^2)^{1/2}, \quad (1)$$

where t is the appropriate quantile of the t distribution.

The set of values of θ_0 for which (1) does not hold is a confidence interval for $\theta = \psi_1/\psi_2$. If the confidence interval is constructed by inverting the test in this way, the interval inherits the good properties of the test. By solving the quadratic equation related to (1), the confidence interval is found to be

$$\begin{aligned} &(\hat{\psi}_2^2 - t^2s_2^2)^{-1}[\hat{\psi}_1\hat{\psi}_2 - t^2rs_1s_2 \\ &\pm t\sqrt{\hat{\psi}_1^2s_2^2 - 2r\hat{\psi}_1\hat{\psi}_2s_1s_2 + \hat{\psi}_2^2s_1^2 - t^2s_1^2s_2^2(1 - r^2)}. \end{aligned}$$

In terms of the coefficients of variation $c_1 = s_1/\hat{\psi}_1$ and $c_2 = s_2/\hat{\psi}_2$, the confidence interval reads

$$\hat{\theta} \frac{1 - t^2rc_1c_2 \pm t\sqrt{c_2^2 - 2rc_1c_2 + c_1^2 - t^2c_1^2c_2^2(1 - r^2)}}{1 - t^2c_2^2} = (L, U), \quad (2)$$

where $\hat{\theta} = \hat{\psi}_1/\hat{\psi}_2$.

This confidence interval is, of course, valid only when there are two distinct and positive roots of the quadratic equation. An analysis of the inequality (1) leaves us with four distinct cases which may be characterized in terms of the t statistics for the two parameters separately, $T_i = \hat{\psi}_i/s_i$, $i = 1, 2$:

- $T_1 > t$ and $T_2 > t$, Two-sided: (L, U)
- $T_1 > t$ and $T_2 \leq t$, Left-sided: (U, ∞)
- $T_1 \leq t$ and $T_2 > t$, Right-sided: $(0, U)$
- $T_1 \leq t$ and $T_2 \leq t$, Indeterminate: $(0, \infty)$.

If the confidence interval has confidence level $(1 - \alpha)$, then t is the upper $\alpha/2$ quantile of the t distribution. The four cases are then the four possibilities when testing the two hypotheses $H_i: \psi_i = 0$ against $H_i: \psi_i > 0$ ($i = 1, 2$), each at level $\alpha/2$, such that the joint Bonferroni level is α . Neyman (1954) pointed out the unsatisfactory nature of the interval in all but the first of the four cases, and recommended that no inference about θ be made in the other cases. In the minke whale application we are well within the first case.

When $\hat{\psi}_1$ and $\hat{\psi}_2$ are uncorrelated, the confidence interval simplifies by setting $r = 0$.

4. BAYESIAN INFERENCE ABOUT A QUOTIENT

We denote by $p(\psi_1, \psi_2|D)$ the joint posterior density of ψ_1 and ψ_2 given the data D . The posterior density of θ is then simply

$$p(\theta|D) = \int_0^\infty \psi_2 p(\theta\psi_2, \psi_2|D) d\psi_2. \quad (3)$$

If $p(\theta|D)$ does not have an analytic form, the right side of (3) is a one-dimensional integral that can be readily and quickly evaluated numerically. The cumulative posterior distribution also lends itself readily to numerical evaluation by summing up values of $p(\theta|D)$ from (3), and quantiles of the posterior distribution can be read off easily.

A simple analytic approximation to Equation (3) is available when $p(\psi_1, \psi_2|D)$ is approximately bivariate normal, and the approximating normal distribution of ψ_2 has almost no mass below zero. This case is of considerable practical interest because posterior distributions are asymptotically normal under fairly general conditions (Heyde and Johnstone 1979; Walker 1969). Here ψ_2 is known to be positive and so its true posterior distribution is entirely concentrated on the positive numbers; if the normal approximation is adequate it is likely to place very little mass on the negative numbers.

Suppose that $E[\psi_i|D] = \hat{\psi}_i$, $\text{var}(\psi_i|D) = \sigma_i^2$ ($i = 1, 2$) and $\text{corr}(\psi_1, \psi_2|D) = \rho$. We assume that $\hat{\psi}_2/\sigma_2$ is large enough that the normal distribution assigns negligible probability to negative values of ψ_2 ; otherwise the normal approximation is unlikely to be useful. Then

$$P[\theta \leq \theta] = P\left[\frac{\psi_1}{\psi_2} \leq \theta\right] = P[\psi_1 - \theta\psi_2 \leq 0], \quad (4)$$

since ψ_2 is positive. Now, $E[\psi_1 - \theta\psi_2|D] = \hat{\psi}_1 - \theta\hat{\psi}_2$ and $\text{var}(\psi_1 - \theta\psi_2|D) = \sigma_1^2 - 2\theta\rho\sigma_1\sigma_2 + \theta^2\sigma_2^2 = V(\theta)$ say. Thus (4) yields

$$P[\theta \leq \theta|D] \approx \Phi\left(-\frac{\hat{\psi}_1 - \theta\hat{\psi}_2}{\sqrt{V(\theta)}}\right), \quad (5)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. Differentiation of (5) with respect to θ yields

$$p(\theta|D) \approx \frac{a + b\theta}{V(\theta)^{3/2}} \phi\left(-\frac{\hat{\psi}_1 - \theta\hat{\psi}_2}{\sqrt{V(\theta)}}\right), \quad (6)$$

where $a = (\hat{\psi}_2\sigma_1^2 - \hat{\psi}_1\rho\sigma_1\sigma_2)$, $b = (\hat{\psi}_1\sigma_2^2 - \hat{\psi}_2\rho\sigma_1\sigma_2)$ and ϕ is the standard normal density. Equations (5) and (6) readily yield Bayesian estimation intervals and measures of location of the posterior distribution such as the posterior median and mode.

Note that $\theta^2 p(\theta|D)$ tends to a constant as $\theta \rightarrow \infty$, so that θ has no finite posterior moments of order greater than or equal to one. Thus the posterior mean just fails to exist, and the posterior variance does not exist. Also, as $\theta \rightarrow \infty$, the right side of Equation (5) tends to $\Phi(\hat{\psi}_2/\sigma_2)$ rather than to one; the difference is negligible if the normal approximation is good. The approximate posterior density given by Equation (6) is negative for

some values of θ , namely those greater than θ_{crit} when $\rho < d$ and those less than θ_{crit} when $\rho \geq d$, where

$$\theta_{\text{crit}} = \frac{\sigma_1 d \rho - 1}{\sigma_2 d - \rho}, \quad (7)$$

and $d = (\hat{\psi}_1/\sigma_1)/(\hat{\psi}_2/\sigma_2)$. In practical situations such as the minke whale one, this is typically not a problem because θ_{crit} is usually negative, or at least well below the lower bound of any reasonable posterior interval for θ . When ρ is close to one, however, there can be problems because then θ_{crit} is close to σ_1/σ_2 ; if $\rho < d$ and σ_1/σ_2 is large, or if $\rho \geq d$ and σ_1/σ_2 is small, the approximation may not be satisfactory. When $\rho \geq d$ or when ρ is close to one, it seems wise to calculate θ_{crit} using Equation (7) to check that the density is positive for the entire range of plausible values of θ . In the minke whale example, $\rho = 0$, $d = .8$, and $(\hat{\psi}_2/\sigma_2) = 8$, so that $\theta_{\text{crit}} = -126,350$ and there is no problem.

The Bayesian approach also allows the incorporation of uncertainty about model assumptions as represented by the values of $\lambda = (\hat{\psi}_1, \sigma_1, \hat{\psi}_2, \sigma_2, \rho)$. This can arise because inferences about ψ_1 and ψ_2 are typically based on modeling assumptions, and different models will lead to different inferences, as represented by λ . This is done by integration in the usual way. Suppose that uncertainty about λ given all available information is represented by a density $p(\lambda|D)$. Then we have

$$p(\theta|D) = \int p(\theta|\lambda, D)p(\lambda|D) d\lambda. \quad (8)$$

This approach was taken by the IWC to resolve disagreement in the case of Northeast Atlantic minke whale abundance, as we describe in the next section.

5. APPLICATION: ESTIMATING MINKE WHALE ABUNDANCE

In 1990, the IWC undertook a comprehensive assessment of the various North Atlantic minke whale stocks as part of its review of the worldwide moratorium on commercial whaling. Before this assessment, the IWC abundance estimate of North Atlantic minke whales was 19,112 (IWC 1989). This estimate was termed "provisional" because it was suspected of downward bias. Based on the results presented to the IWC (Schweder et al. 1991) and referred to in Section 2, with a point estimate of

$$\hat{\theta} = 34,800/.427 = 81,500$$

and with a standard error of 18,000, the hypothesis that $\theta = 19,112$ must be rejected.

A two-sided confidence interval was found by the Fieller technique of Section 3. There is no problem with negative values in the normal approximation because $\hat{\psi}_1$ and $\hat{\psi}_2$ are both at least six standard errors greater than zero. The 95% confidence interval is (55,000, 125,000). Here we have assumed $\hat{\psi}_1$ and $\hat{\psi}_2$ to be independent since they are based on separate experiments. We have also assumed the estimates to be approximately normally distributed, which they are to a good approximation, and to have "precisely" estimated stan-

dard errors. In constructing the confidence interval the standard normal cutoff point of 1.96 was used, and the interval is approximate.

One puzzling aspect of the Fieller interval is the following. If one takes no account of the uncertainty about the denominator and conditions on $\psi_2 = \hat{\psi}_2 = .427$, the resulting interval is (56,000, 107,000). Thus taking account of the uncertainty about the denominator reduces the lower bound only slightly, but stretches the upper bound considerably. It is the hyperbolic relation between the quotient and the denominator that causes this asymmetric effect.

The Bayesian analysis goes as follows. The posterior density of θ given by (6) and based on the data of Schweder et al. (1991) is shown as the dotted-dashed line in Figure 1. In the Bayesian context, a point estimator arises only as the solution to a decision problem corresponding to a specific loss function. For this example, it seems reasonable that *relative* (or percentage) error is what we should be trying to minimize, suggesting the loss function $L(\hat{\theta}) = ((\hat{\theta} - \theta)/\theta)^2$. The value minimizing this expected loss is the point estimator

$$\hat{\theta}_{\text{MRE}} = \frac{E[\theta^{-1}|D]}{E[\theta^{-2}|D]}, \quad (9)$$

see Raftery (1988). This and various other summaries of the posterior distribution are shown in Table 1. Note that the posterior moments in Equation (9) do exist. Another possible loss function was suggested by Zellner (1978) for a ratio of regression parameters, but the relative squared error loss function seems satisfactory in the present context of population size estimation.

When the results of Øien (1991) and Schweder et al. (1991) were discussed by the Scientific Committee of IWC in its 1990 meeting, some members felt that bias in the abundance estimate may have been introduced by systematic errors in the eye-measured distances from observer to primary whale observation. They referred to Figure 2, which gives a scatterplot of eye-measured distances versus triangulated distances. The latter are calculated by assuming that the angles and the two distances identified in Figure 3 for surfacings observed from both ships are measured without error. Against

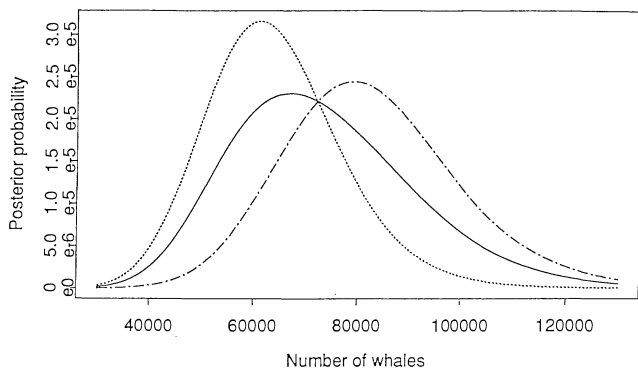


Figure 1. Posterior Distribution of the Number of Whales. The dotted-dashed line shows the "uncorrected" posterior density, the dotted line shows the "corrected" posterior density, while the solid line shows the posterior density that was agreed by the IWC Scientific Committee in 1990.

Table 1. Summaries of the Three Posterior Distributions

Summary	"Uncorrected"	"Corrected"	Agreed
$\hat{\theta}_{\text{MRE}}$	76,000	59,000	65,000
Posterior mode	79,000	61,000	67,000
Posterior median	81,500	63,000	71,500
95% Symmetric interval			
Lower bound	53,000	41,000	43,500
Upper bound	121,000	93,500	114,000
95% HPD region			
Lower bound	50,500	39,000	42,000
Upper bound	117,500	91,000	110,500

this interpretation of Figure 2, it was argued in the Scientific Committee that the angles are subject to measurement error, error caused by the rolling of the ships, and bias due to the reaction delay of the observer, and that the variability in Figure 2, which hints at underestimation of distances, was to be expected in the scatter of observed versus triangulated distances even when there was no bias in the distance measurements.

In an exercise, which some members understood as an attempt at correcting the abundance estimates of Schweder et al. (1991), while other members understood it as a sensitivity analysis, hypothetical "corrections" were put forward for both $\hat{\psi}_1$ and $\hat{\psi}_2$. For $\hat{\psi}_1$,

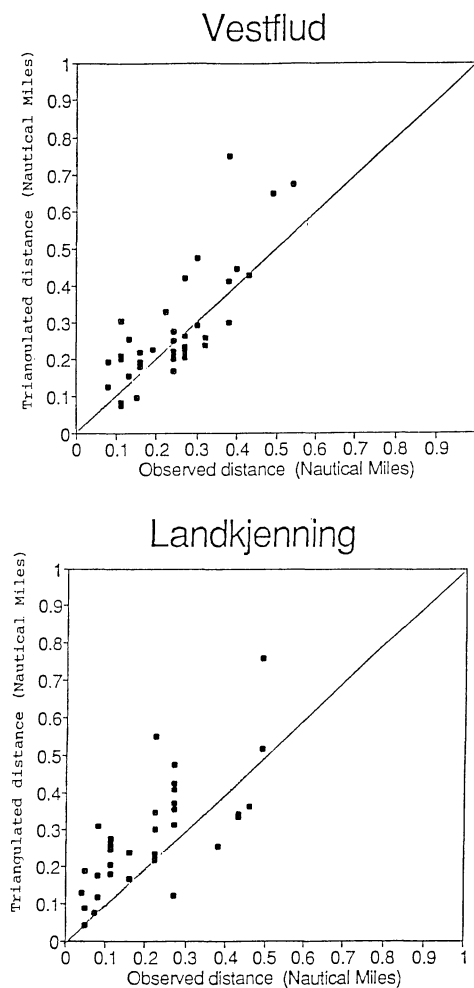


Figure 2. Triangulated Versus Observed Distances to Duplicate Sightings From the Vessels Landkjenning and Vestflud.

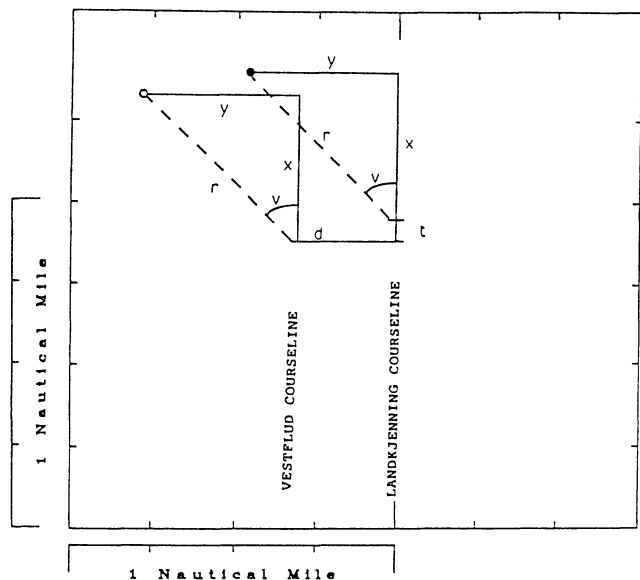


Figure 3. Distances and Angles for a Duplicate Sighting. The triangulated position is far distant and is not shown. Open circle, sighted by Vestflud; closed circle, sighted by Landkjenning.

distances were calibrated by the regression of triangulated distances on observed ones. For $\hat{\psi}_2$ the calibration was done by the regression the other way around, of observed distances on triangulated ones. Because regressing y on x is different from regressing x on y , these two calibrations acted very differently. In the one case, the observers were assumed to overestimate long distances while in the other they were assumed to underestimate them. This is self-contradictory, since the ordinary shipboard survey and the parallel ship experiment were conducted under similar circumstances and with the same observers. This paradox did not bother those members of the Scientific Committee who wanted to use the “calibration” exercise to correct the original abundance estimate. The effect of the “correction” was to reduce $\hat{\psi}_1$ and to increase $\hat{\psi}_2$, and the effect on the abundance estimate was substantial. In Schweder et al. (1991), which was revised after the meeting, this method of “correcting” was shown to be invalid.

In any event, the Scientific Committee had to come up with some estimate of North East Atlantic minke whale abundance. A compromise was sought between those who wanted to “correct” the estimates of Schweder et al. (1991) and those who found the uncorrected numbers to be the best possible. The compromise that was accepted was to follow a Bayesian approach incorporating model uncertainty using Equation (8). With λ as defined in the context of the integration of Equation (8), a two-point distribution with probability 1/2 at each point was chosen for $p(\lambda|D)$. One point represented those who favored the “uncorrected” abundance estimates, namely

$$\lambda_1 = (34,800, 5,500, .427, .0542, 0),$$

and the other point represented those who wanted the abundance estimate to be “corrected” downwards,

namely

$$\lambda_2 = (26,900, 4,250, .427, .0542, 0).$$

The “correcters” proposed only to change $\hat{\psi}_1$ and its standard error—to avoid “overcorrection.” The resulting combined posterior density is shown as the solid line in Figure 1, together with the “corrected” and “uncorrected” posterior densities. Summaries of all three posterior distributions are shown in Table 1.

The Scientific Committee adopted the symmetric 95% Bayesian estimation interval, (43,500, 114,000), as its interval estimate of the number of North East Atlantic minke whales (IWC 1991; Raftery, Buckland, Cooke, and Schweder 1991). This is the interval with the .025 and .975 quantiles as limits, and not the highest posterior density interval which the Scientific Committee did not find useful. The Scientific Committee did not, however, agree to put forward a point estimate of abundance. Some members argued that a point estimate is uninterpretable in the presence of such a highly dispersed posterior, and they opposed choosing the median or any other measure of location of the posterior distribution as an agreed abundance estimate. In the IWC meeting proper, following that of the Scientific Committee, it was decided, however, that the comprehensive assessment of North Atlantic minke whales was not complete without an agreed point estimate of abundance, and that the Scientific Committee would have to produce an agreed point estimate at its 1991 meeting.

EPILOGUE

In 1990, additional data were gathered to obtain a better estimate of ψ_2 and to study the bias and errors in eye-measured distances from the observer to the surfacing whale. A revised estimate of ψ_2 was presented to the 1991 meeting of the Scientific Committee. This time the Committee accepted a point estimate of abundance but it did not accept a confidence interval. A thorough discussion of method took place, however. Based on recommendations given in 1991, the estimate of ψ_2 was recomputed by a new method which allowed the data on measurement errors to be incorporated in the estimation (Schweder et al. 1992). This estimate was presented to the Scientific Committee in 1992 and this time around a point estimate of 86,736 with a confidence interval of (60,736, 117,449) was accepted (IWC 1993).

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