Math 493

Midterm Examination 1 – October 6, 2010

Name ________________________________

General Instructions: Please answer the following, showing all your work, and without the aid of books or notes.

1. (1 points each) True/False. Please read the statements carefully, as no partial credit will be given.

   (a) ____ If A, B are any events in some sample space, then

   $$P(A \cap B) = P(A) + P(B) - P(A \cup B).$$

   (b) ____ The function $m(j) = \frac{1}{j!}$ is a probability distribution function on $\Omega = \mathbb{N}$.

   (c) ____ If A and B are disjoint events, then

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap C) - P(B \cap C).$$

   (d) ____ The following function is a probability density function:

   $$f(x) = \begin{cases} \frac{1}{x^2} & \text{for } x \geq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

   (e) ____ If A, B, and C are independent events, then

   $$P(A \cup B \cup C) = P(A) + P(B) + P(C).$$

   (f) ____ If X is an exponentially distributed continuous random variable, then $P(X > 0) = \frac{1}{2}$.

   (g) ____ If Y is a uniformly distributed continuous random variable on $[0, \pi]$ then $P(X = e) = 0$.

   (h) ____ If Z is a uniformly distributed continuous random variable on $[0, \pi]$, then the cumulative distribution function for Z is

   $$F(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{x}{\pi} & \text{for } 0 \leq x \leq \pi \\ 1 & \text{for } x > \pi. \end{cases}$$

2. (4 points each)

   (a) A deck is shuffled into uniformly random permutation, and a whist hand (13 cards) is dealt out. Find the probability that the hand consists of 4 cards of one suit, and 3 cards of each of the other three suits.

   (b) You roll a die 4 times. What is the probability that no two of the resulting numbers are the same?

   (c) You begin repeatedly rolling a die. Let X be the number of rolls before you first get a 6. Find a probability distribution $m(j)$ for X.

3. (a) (4 points) Let X be a continuous real-valued random variable with a (Riemann) density function $f(x)$. Suppose that E is an interval. Show that

$$P(X \in E^c) = 1 - P(X \in E).$$

(Note: We proved the discrete analogue of this in class.)

(b) (6 points) Let X and Y be continuous uniform random variables on $[0, 1]$. Find $P(|2X - Y| < 1)$. (A picture may be helpful!)