Math 535, Homework 1, due Oct 1

(1) If Δ is a finite simplicial complex on vertex set V, and $S \subseteq V$, then the *induced subcomplex* on S (denoted $\Delta[S]$) is the simplicial complex with all faces of Δ that are contained in S.
Characterize the simplicial complexes Δ such that $H_i(\Delta[S]; \mathbb{Z}) = 0$ for all $i > 0$ and all subsets $S \subseteq V(\Delta)$.

(2) Let L be a finite lattice, and let $\Gamma(L)$ be the simplicial complex with vertex set the atoms of L, and faces all subsets of atoms $x_1, \ldots, x_k$ such that $x_1 \vee \cdots \vee x_k \neq \hat{1}$.
Show that $\Gamma(L)$ is homotopy equivalent to $|L|$.

(3) A CW-complex is *regular* if every attaching map $D^n \to X^{n-1}$ is a homeomorphism. The face poset $L(X)$ of a regular CW-complex X consists of all cells of X, ordered by inclusion. Show that $|L(X)|$ is homeomorphic to X.

(4) Find a poset P with $|P|$ a Möbius strip. Find $\mu_P(x, y)$ for every isomorphism type of interval in $[x, y]$.

(5) For Δ a simplicial complex on vertex set V, let $\Delta^\vee$ be the simplicial complex with vertex set V and <u>non</u>-faces consisting of $\{V \setminus \sigma \,:\, \sigma \in \Delta\}$.
 (a) Show that $(\Delta^\vee)^\vee = \Delta$.
 (b) Show that $\tilde{H}_i(\Delta; \mathbb{Z}) \cong \tilde{H}^{n-i-3}(\Delta^\vee; \mathbb{Z})$, where $n = |V|$.
 Hint: Embed in a simplex and apply Alexander duality!
 (c) Find $\chi(\Delta^\vee)$.