Math 535, Homework 2, due Nov 8

(1) Prove that if a lattice L has a (not necessarily maximal) chain

$$\hat{0} = m_0 < m_1 < \cdots < m_k = \hat{1}$$

with each m_i modular in L, then $\tilde{H}_i(|L|)$ vanishes for $i < k-2$.

(2) Let G be a finite group, and $\mathcal{S}(G)$ be the subnormal subgroup lattice:

(a) Show that $\mathcal{S}(G)$ is a lattice.
(b) Show that $\mathcal{S}(G)$ is lower semi-modular.
(c) Is $\mathcal{S}(G)$ geometric for all G? Prove or find a counterexample.

(3) (a) Show that if L is a semimodular lattice, then every coatom of L^* is modular. (That is, show every atom of L is dual modular.)
(b) If L is a geometric lattice, describe the descending chains of the associated EL-labeling with respect to atom ordering $a_1, \ldots, a_s$.

(4) Let P_n be the graph consisting of a path with n-vertices. Describe the homotopy type of the independence complex of P_n.

(5) The *n-dimensional cross polytope* X_n is the independence complex of $n+1$ disjoint edges.

(a) Show that the face lattice $L(X_n)$ has an EL-labeling.
(b) Conclude that $X_n \simeq S^n$.

(6) Characterize the graphs G for which the lattice of flats of the cycle matroid is supersolvable.