Math 132
Midterm Examination 3 – April 4, 2012

6 multiple choice, 4 long answer. 100 points.

General Instructions: Please answer the following, without use of calculators. You
may refer to a 3x5 card, but no other notes. Part I of the exam is multiple choice,
while Part II is long answer.

Part I Instructions: If you do not have a <u>pencil</u> to fill out your answer card, please ask
to borrow one from your proctor. Write your Student ID number on the six blank lines
on the top of your answer card, and shade in the corresponding bubbles to the right
of each digit.
Fill in the bubble corresponding to each of the following 6 questions. Each is worth 4
points. On Part I, no partial credit will be given.

1. The geometric series $\sum_{i=1}^{\infty} \frac{(-1)^{i+1}}{3^i}$ converges to:

   (a) 0

   (b) $\frac{1}{6}$

   (c) $\frac{1}{4}$

   (d) $\frac{1}{3}$

   (e) $\frac{1}{2}$

   (f) $\frac{2}{3}$

   (g) $\frac{3}{4}$

   (h) $\frac{5}{6}$

   (i) 1

   (j) Does not converge – oscillates.

   (k) Does not converge – diverges to ∞.

2. Evaluate $\int_0^8 \frac{1}{\sqrt[3]{x}}\,dx$.

   (a) 0
   (b) 1
   (c) 2
   (d) 3
   (e) 4
   (f) 5
   (g) 6
   (h) 7
   (i) 8
   (j) 16
   (k) ∞

3. Evaluate $\int_4^\infty \frac{1}{x^{5/2}}\,dx$.

   (a) $-\infty$
   (b) 0
   (c) $\dfrac{1}{20}$
   (d) $\dfrac{1}{12}$
   (e) $\dfrac{1}{3}$
   (f) $\dfrac{1}{2}$
   (g) $\dfrac{2}{3}$
   (h) $\dfrac{3}{4}$
   (i) $\dfrac{5}{4}$
   (j) ∞
   (k) Does not exist/undefined/diverges.

4. Evaluate $\int_{-2}^{2} \frac{1}{x^2} \, dx$.

   (a) -2

   (b) $-2 \ln 2$

   (c) -1

   (d) $-\ln 2$

   (e) 0

   (f) $\ln 2$

   (g) 1

   (h) $2 \ln 2$

   (i) 2

   (j) Does not exist/undefined/diverges.

5. Evaluate $\int_0^{\pi/2} \sin 2\theta \cos \theta \, d\theta$.

   (a) 0

   (b) $\dfrac{1}{6}$

   (c) $\dfrac{1}{4}$

   (d) $\dfrac{1}{3}$

   (e) $\dfrac{1}{2}$

   (f) $\dfrac{2}{3}$

   (g) $\dfrac{3}{4}$

   (h) $\dfrac{5}{6}$

   (i) 1

   (j) ∞

6. The sequence $\dfrac{n^4}{n!}$ converges (as $n \to \infty$) to:

   (a) $-\infty$

   (b) -4

   (c) -2

   (d) -1

   (e) 0

   (f) 1

   (g) 2

   (h) 4

   (i) ∞

   (j) Does not exist/undefined/diverges.

Part II Instructions: Answer the following on the exam sheet, showing all your work. Correct answers without correct supporting work may not receive full credit. You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Please put your name and student id number on **each page** of Part II now.

1. Exact evaluation of improper integrals and series

(a) (6 points) Evaluate $\displaystyle\sum_{i=1}^{\infty} \frac{2^i + 3^i}{4^i}$.

(b) (6 points) For what values of x does $\displaystyle\sum_{i=0}^{\infty} x^i$ converge? When it converges, what does it converge to?

(c) (6 points) Evaluate $\displaystyle\int_0^{\infty} xe^{-x}\, dx$.

(d) (6 points) Using partial fractions, evaluate $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$.

2. Integration techniques

(a) (6 points) Evaluate $\displaystyle\int \frac{z+4}{z^3+z}\,dz$.

(b) (6 points) Evaluate $\displaystyle\int \frac{-9x^2-3x+6}{x^4-5x^2+4}\,dx$.

(c) (6 points) Evaluate $\displaystyle\int \frac{e^{3x}}{\sqrt{1-e^{2x}}}\,dx$.

(d) (5 points) Evaluate $\displaystyle\int_0^1 \frac{1}{(3-x^2)^{3/2}}\,dx$.

3. Series convergence.

Determine whether each of the following series converges or diverges.

(a) (6 points) $\displaystyle\sum_{i=1}^{\infty} \frac{2(i-1)(i-2)}{3(i+1)(i+2)}$

(b) (6 points) $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i^{3/2} + \sin^2 i}$

(c) (6 points) $\displaystyle\sum_{i=1}^{\infty} \frac{2}{i + \ln i}$

4. Comparison tests for integrals

Use a test for convergence for each problem on this page. (Don't try to find anti-derivatives!)

(a) (5 points) Show that $\displaystyle\int_1^\infty \frac{1}{x^2 + \sqrt{x}}\,dx$ converges.

(b) (5 points) Show that $\displaystyle\int_0^1 \frac{1}{x^2 + \sqrt{x}}\,dx$ converges.

(c) (1 point) Conclude that $\displaystyle\int_0^\infty \frac{1}{x^2 + \sqrt{x}}\,dx$ converges.