

Name: \_\_\_\_\_

NO CALCULATORS

ONE  $4 \times 6$  NOTECARD ALLOWED

SHOW WORK &amp; SIMPLIFY ANSWERS

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \tan x \sec x \, dx = \sec x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \cot x \csc x \, dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2+x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{x\sqrt{x^2-a^2}} \, dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \dots$$

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \dots$$

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \dots$$

$$\ln(1-x) = -x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \dots$$

1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/10
Total	/100

1. Evaluate the following definite integrals:

(a)  $\int_0^{\pi/3} 2x \sin 2x \, dx$

(i)  $\frac{\pi}{2} - \frac{1}{4}$     (ii)  $\frac{3\pi}{4} - \frac{1}{6}$     (iii)  $\frac{\pi}{3} + \frac{\sqrt{3}}{2}$     (iv)  $\frac{\pi}{6} + \frac{\sqrt{3}}{4}$     (v)  $\frac{\pi}{3} + \frac{\sqrt{2}}{2}$

(b)  $\int_1^{\infty} \frac{\ln y}{y^2} \, dy$

(i) 0    (ii)  $\frac{1}{2}$     (iii) 1    (iv)  $\frac{3}{2}$     (v)  $\infty$

2. Evaluate the following definite integrals:

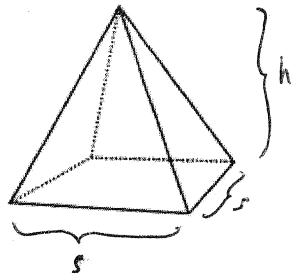
(a)  $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

(i)  $2e^2 - 2e$     (ii)  $(e^2 - e)/2$     (iii)  $2e^4 - 2e$     (iv)  $(e^4 - e)/2$     (v)  $\infty$

(b)  $\int_{-\pi/2}^{\pi/2} \frac{\cos y}{\sin^2 y} dy$

(i) 0    (ii) 2    (iii) -2    (iv)  $\infty$     (v)  $-\infty$

3. Find the volume of a pyramid whose height is  $h$  and whose base is a square with side length  $s$ .



- (i)  $\frac{sh}{4}$     (ii)  $\frac{s^2h}{3}$     (iii)  $\frac{s}{4h^2}$     (iv)  $\frac{sh^2}{2}$     (v)  $\frac{s^2}{2h}$

4. Find the general indefinite integrals.

(a)  $\int \sqrt{6x - x^2} dx$

(b)  $\int \frac{y^2-2}{y^2-1} dy$

5. Find  $\int \frac{1}{z^3 - 4z^2 + 5z} dz$ .

6. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent.

(a)  $\sum_{n=2}^{\infty} \frac{1}{n^2 \ln n}$

(i) absolutely convergent      (ii) conditionally convergent      (iii) divergent

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{e^{1/n}}$

(i) absolutely convergent      (ii) conditionally convergent      (iii) divergent

(c)  $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+e^{1/n}}$

(i) absolutely convergent      (ii) conditionally convergent      (iii) divergent

7. Determine the interval of convergence of each power series.

(a)  $\sum_{n=2}^{\infty} \frac{1}{\ln n} (x - 3)^n$

(i)  $[2, 4]$     (ii)  $[2, 4)$     (iii)  $(1, 5)$     (iv)  $(1, 5]$     (v)  $(-\infty, \infty)$

(b)  $\sum_{n=0}^{\infty} \frac{1}{n!} (x - 2)^n$

(i)  $[1, 3]$     (ii)  $[1, 3)$     (iii)  $(0, 4)$     (iv)  $(0, 4]$     (v)  $(-\infty, \infty)$



8. Use Taylor's formula to find the first five nonzero terms of the Taylor series for the function  $f(x) = \sqrt{x}$  expanded about the point  $x = 1$ .

9. Express as infinite series.

(a)  $\int \frac{e^x - 1}{x} dx$  (include up to the  $x^3$  term)

(b)  $x \cos^2 x$  (include up to the  $x^5$  term)

10. Solve the differential equations.

(a)  $y' = x^2 e^y$ ,  $y(1) = 0$

(b)  $y' = \frac{2y}{x} + x^5$  (give the general solution)