Math 132
Worksheet 1 – January 24, 2012

Name _____

1. (a) Give an example of a partition of $[0, 1]$ into 4 parts which are not uniform (i.e., not all of the same size). Set up the Riemann sum corresponding to this partition for the function $f(x) = x^2$.

   (b) Give a rule for partitioning $[0, 1]$ into n parts, not all of the same size.

2. A sequence of Riemann sums is created from the following data: the partition (into n parts) is given by $x_k = -\pi + \frac{2\pi k}{n}$, the selected point on the kth part is given by $c_k = x_k$, and the function is $\cos x$.

   (a) What rule was used to select the points c_k?

   (b) Write down the corresponding definite integral.

3. Recall that a rational number is one that has a finite or repeated decimal expansion (examples: $\frac{3}{2} = 1.5$, $\frac{1}{3} = 0.\overline{3}$), and an irrational number is has an infinite non-repeating decimal expansion (examples: π, e). It is not hard to see that every nontrivial interval contains both rational and irrational numbers.
In Example 1 on p308 of your text, the authors discuss the function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{otherwise,} \end{cases}$$

which is continuous nowhere, and discuss two rules for creating Riemann sums for it.

(a) Rule U picks c_k to be any rational point on the kth part. Draw a picture of the corresponding Riemann sum.

(b) Rule L picks c_k to be any irrational point on the kth part. Draw a picture of the corresponding Riemann sum.

(c) Conclude that the definite integral $\int_0^1 f(x)\,dx$ doesn't make sense. What does this say about area?

(d) Calculate the Riemann sum corresponding to uniform partitions and the right endpoint rule for $\int_0^1 f(x)\,dx$ and $\int_0^\pi f(x)\,dx$. Do the limits converge? How does this relate to part (c)?