

Math 132

Worksheet 13 – April 24, 2012

Name _____

1. In 2-3 sentences, explain why if $f(x)$ has a power series representation around 0, then $f(x^2)$ also has a power series representation around 0.

2. Solving a recurrence using power series.

- (a) Suppose we are interested in the sequence given by the recurrence $c_i = c_{i-1} + 2c_{i-2}$ for $i \geq 2$, with initial conditions $c_0 = 0$ and $c_1 = 1$. List out the first 6 terms of this sequence.

- (b) Using polynomial long division, explain why if $\sum_{i=0}^{\infty} c_i x^i$ is a power series representation for $\frac{x}{1-x-2x^2}$, then c_i is the sequence from part (a).

$$1 - x - 2x^2 \overline{) \begin{array}{r} x + \\ x \\ \hline x - x^2 - 2x^2 \end{array}}$$

(c) Using partial fractions, find an explicit power series representation for $\frac{x}{1-x-2x^2}$.

(d) On what interval does your series from (c) converge to $\frac{x}{1-x-2x^2}$?

(e) By combining parts (b) and (c), find an explicit formula for the sequence from part (a).