

Homework #1 Solutions

1. $(x - 4y = -10) \cdot -2 \Rightarrow -2x + 8y = 20$

$$3x - 8y = 30$$

$$\begin{array}{r} -2x + 8y = 20 \\ \hline \end{array}$$

$$x = 50$$

$$4y = 60 \Rightarrow y = 15$$

$$(x, y) = (50, 15)$$

2. $3x + 6y + 9z = -18$

$$2x - 3y - 4z = 15$$

$$3x + 4y + 5z = -8$$

$$\begin{array}{r} 3x + 6y + 9z = -18 \\ - (3x + 4y + 5z = -8) \\ \hline 2y + 4z = -10 \end{array}$$

$$-(6x - 9y - 12z = 45)$$

$$6x + 8y + 10z = -16$$

$$\hline 17y + 22z = -61$$

$$\begin{array}{r} 34y + 44z = -122 \\ - (22y + 44z = -110) \\ \hline 12y = -12 \end{array}$$

$$2x = 4$$

$$x = 2$$

$$y = -1$$

$$4z = -8$$

$$z = -2$$

$$(x, y, z) = (2, -1, -2)$$

3. $x + 2y + z = 4$

$$x + y + 2z = 8$$

$$-(x + 4y - z = 5)$$

$$\hline (-3y + 3z = 3) \cdot -1/3$$

$$y - z = -1$$

$$\begin{array}{r} x + 2y + z = 4 \\ - (x + y + 2z = 8) \\ \hline y - z = -4 \end{array}$$

$-1 \neq -4$ inconsistent

$$\text{no solution}$$

4. $a + b = 8$

$$a - b = 6$$

$$\hline 2a = 14$$

$$a = 7$$

$$b = 1$$

$$c - d = 2$$

$$c + d = 4$$

$$\hline 2c = 6$$

$$c = 3$$

$$d = 1$$

$$(a, b, c, d) = (7, 1, 3, 1)$$

$$5. A = \begin{bmatrix} 2 & 4 \\ 4 & -1 \\ 5 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix}$$

$$i) A \text{ is } 3 \times 2$$

$$ii) AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \\ 5 & 7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 2(1)+4(5) & 2(3)+4(-1) \\ 4(1)-1(5) & 4(3)-1(1) \\ 5(1)+7(5) & 5(3)+7(-1) \end{bmatrix} = \begin{bmatrix} 22 & 2 \\ -1 & 13 \\ 40 & 8 \end{bmatrix}$$

$$iii) A^T = \begin{bmatrix} 2 & 4 & 5 \\ 4 & -1 & 7 \end{bmatrix}$$

$$6. A = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 6 & 14 \\ -1 & -3 & -7 \end{bmatrix}$$

$$i) A^T = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 6 & -3 \\ 7 & 14 & -7 \end{bmatrix}$$

$$ii) A^2 = \begin{bmatrix} 1 & 3 & 7 \\ 2 & 6 & 14 \\ -1 & -3 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 7 \\ 2 & 6 & 14 \\ -1 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 6 & -3 \\ 7 & 14 & -7 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 7 \\ 2 & 6 & 14 \\ -1 & -3 & -7 \end{bmatrix} = \begin{bmatrix} 6 & 18 & 42 \\ 18 & 54 & 126 \\ 42 & 126 & 294 \end{bmatrix}$$

$$A^2 \neq A^T A$$

$$iii) \text{tr}(A^2) = 0 + 0 + 0 = 0$$

$$\text{tr}(A^T A) = 6 + 54 + 294 = 354$$