Homework #9 Solutions

5.1

1. Let $u = [u_1 \ \ u_2]$ and $v = [v_1 \ \ v_2]$

a) $L(u+v) = [u_1+v_1+1 \quad u_2+v_2 \quad u_1+v_1+u_2+v_2]$

$L(u)+L(v) = [u_1+1 \quad u_2 \quad u_1+u_2] + [v_1+1 \quad v_2 \quad v_1+v_2]$

$= [u_1+v_1+2 \quad u_2+v_2 \quad u_1+v_1+u_2+v_2]$

$\therefore L(u+v) \neq L(u)+L(v)$

$\boxed{\text{No, } L \text{ is not a linear transformation}}$

b.) $L(u+v) = [u_1+v_1+u_2+v_2 \quad u_2+v_2 \quad u_1+v_1-u_2-v_2]$

$L(u)+L(v) = [u_1+u_2 \quad u_2 \quad u_1-u_2] + [v_1+v_2 \quad v_2 \quad v_1-v_2]$

$= [u_1+v_1+u_2+v_2 \quad u_2+v_2 \quad u_1+v_1-u_2-v_2]$

$\therefore L(u+v) = L(u)+L(v)$

$L(cu) = [cu_1+cu_2 \quad cu_2 \quad cu_1-cu_2]$

$cL(u) = c[u_1+u_2 \quad u_2 \quad u_1-u_2]$

$= [cu_1+cu_2 \quad cu_2 \quad cu_1-cu_2]$

$\therefore L(cu) = cL(u)$

$\boxed{\text{Yes, } L \text{ is a linear transformation}}$

6. a.) $L\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\-1\end{bmatrix}$, $L\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\0\end{bmatrix}$ $\boxed{\text{Matrix representing } L: \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}}$

b.) $L\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$, $L\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}k\\1\end{bmatrix}$

$\boxed{\text{Matrix representing } L: \begin{bmatrix}1 & k\\0 & 1\end{bmatrix}}$

c.) $L\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}k\\0\\0\end{bmatrix}$, $L\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\k\\0\end{bmatrix}$

$L\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\0\\k\end{bmatrix}$ $\boxed{\text{Matrix representing } L: \begin{bmatrix}k & 0 & 0\\0 & k & 0\\0 & 0 & k\end{bmatrix}}$

10. a.) $\begin{bmatrix}0 & -1 & 2\\-2 & 1 & 3\\1 & 2 & -3\end{bmatrix}\begin{bmatrix}2\\-3\\1\end{bmatrix} = \begin{bmatrix}5\\-4\\-7\end{bmatrix}$ $\boxed{L\left(\begin{bmatrix}2\\-3\\1\end{bmatrix}\right) = \begin{bmatrix}5\\-4\\-7\end{bmatrix}}$

b.) $\begin{bmatrix}0 & -1 & 2\\-2 & 1 & 3\\1 & 2 & -3\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix} = \begin{bmatrix}-u_2+2u_3\\-2u_1+u_2+3u_3\\u_1+2u_2-3u_3\end{bmatrix}$ $\boxed{L\left(\begin{bmatrix}u_1\\u_2\\u_3\end{bmatrix}\right) = \begin{bmatrix}-u_2+2u_3\\-2u_1+u_2+3u_3\\u_1+2u_2-3u_3\end{bmatrix}}$

13. a) $L(2t^2-5t+3) = L(2t^2) - L(5t) + L(3) = 2L(t^2) - 5L(t) + 3L($
$= 2(t^3+t) - 5(t^2) + 3(1) = 2t^3 - 5t^2 + 2t + 3$

$$\boxed{L(2t^2-5t+3) = 2t^3 - 5t^2 + 2t + 3}$$

b) $L(at^2+bt+c) = L(at^2) + L(bt) + L(c) = aL(t^2) + bL(t) + cL(1)$
$= a(t^3+t) + b(t^2) + c(1)$

$$\boxed{L(at^2+bt+c) = at^3 + bt^2 + at + c}$$

5.2

2. a) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\boxed{\begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ is not in ker } L}$

b) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\boxed{\begin{bmatrix} 2 \\ -1 \end{bmatrix} \text{ is in ker } L}$

c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ 2 & 4 & 6 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow u_1 + 2u_2 = 3 \Rightarrow u_1 = 3 - 2u_2$
$u_2 = r \in \mathbb{R}$

$\boxed{\begin{bmatrix} 3 \\ 6 \end{bmatrix} \text{ is in range } L}$

d) $\left[\begin{array}{cc|c} 1 & 2 & 2 \\ 2 & 4 & 3 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 2 \\ 0 & 0 & -1 \end{array}\right]$ $0 = -1$ inconsistent

$\boxed{\begin{bmatrix} 2 \\ 3 \end{bmatrix} \text{ is not in range } L}$

e) $\left[\begin{array}{cc|c} 1 & 2 & 0 \\ 2 & 4 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \Rightarrow u_1 + 2u_2 = 0 \Rightarrow u_1 = -2u_2$
$u_2 = r \in \mathbb{R}$

$\boxed{\text{ker } L: \begin{bmatrix} -2r \\ r \end{bmatrix} \quad r \in \mathbb{R}}$

f) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 + 2u_2 \\ 2u_1 + 4u_2 \end{bmatrix} = u_1\begin{bmatrix} 1 \\ 2 \end{bmatrix} + u_2\begin{bmatrix} 2 \\ 4 \end{bmatrix} = (u_1 + 2u_2)\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\boxed{\left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\} \text{ spans range } L}$

8. a) $a + b = 0 \Rightarrow a = -b = -c$
$b - c = 0 \Rightarrow b = c$ $\boxed{\{-t^2 + t + 1\}}$

b) range L: $(a+b)t+(b-c) = (a+b)(t)+(b-c)(1) = d(t)+e(1)$

$$\boxed{\{t, 1\}}$$

10. a) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} a+2c & b+2d \\ a+c & b+d \end{bmatrix} - \begin{bmatrix} a+b & 2a+b \\ c+d & 2c+d \end{bmatrix}$

$= \begin{bmatrix} 2c-b & 2d-2a \\ a-d & b-2c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow 2c-b=0 \quad 2d-2a=0$

$a-d=0 \quad b-2c=0$

$\Rightarrow a=d, \; b=2c$

$$\boxed{\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \right\}}$$

b) $L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} 2c-b & 2d-2a \\ a-d & b-2c \end{bmatrix} = (a-d)\begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix} + (b-2c)\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\boxed{\left\{ \begin{bmatrix} 0 & -2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \right\}}$$

22. $a\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}^T + b\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}^T = \begin{bmatrix} a+3b \\ -a+b \\ 2a-b \end{bmatrix}^T$

$L\left(\begin{bmatrix} a \\ b \end{bmatrix}^T\right) = \begin{bmatrix} a+3b \\ -a+b \\ 2a-b \end{bmatrix}^T$

$\boxed{\text{A possible answer: } L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}^T\right) = \begin{bmatrix} u_1+3u_2 \\ -u_1+u_2 \\ 2u_1-u_2 \end{bmatrix}^T}$

5.3

1. a) $L\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \; L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $\quad \boxed{\begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}}$

b) $\left[L\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)\right]_T = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \; \left[L\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right]_T = \begin{bmatrix} -1/2 \\ 3/4 \end{bmatrix}$ $\quad \boxed{\begin{bmatrix} 1 & -1/2 \\ 1 & 3/4 \end{bmatrix}}$

$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\quad \begin{bmatrix} 2 \\ -1 \end{bmatrix} = -1/2\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 3/4\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

c) $\left[L\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)\right]_S = \begin{bmatrix} 3 \\ -4 \end{bmatrix}, \; \left[L\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right)\right]_S = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $\quad \boxed{\begin{bmatrix} 3 & 2 \\ -4 & 4 \end{bmatrix}}$

$\begin{bmatrix} 3 \\ -4 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 4\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\quad \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2\begin{bmatrix} 1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

d) $L\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ -4 \end{bmatrix}, \; L\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\left[L\left(\begin{bmatrix} -1 \\ 2 \end{bmatrix}\right)\right]_T = \begin{bmatrix} -2 \\ 1/2 \end{bmatrix}, \; \left[L\left(\begin{bmatrix} 2 \\ 0 \end{bmatrix}\right)\right]_T = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

$\begin{bmatrix} 3 \\ -4 \end{bmatrix} = -2\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 1/2\begin{bmatrix} 2 \\ 0 \end{bmatrix}$, $\quad \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\quad \boxed{\begin{bmatrix} -2 & 2 \\ 1/2 & 2 \end{bmatrix}}$

e.) $L\left(\begin{bmatrix}1\\2\end{bmatrix}\right)=\begin{bmatrix}5\\0\end{bmatrix}$ ✓

a.) $\begin{bmatrix}1&2\\2&-1\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}5\\0\end{bmatrix}$ ✓

b.) $\begin{bmatrix}1&-1/2\\1&3/4\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix}=\begin{bmatrix}0\\5/2\end{bmatrix} \Rightarrow 0\begin{bmatrix}-1\\2\end{bmatrix}+5/2\begin{bmatrix}2\\0\end{bmatrix}=\begin{bmatrix}5\\0\end{bmatrix}$ ✓

c.) $\begin{bmatrix}1\\2\end{bmatrix}=1\begin{bmatrix}-1\\2\end{bmatrix}+1\begin{bmatrix}2\\1\end{bmatrix}$

$\begin{bmatrix}3&2\\-4&4\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}5\\0\end{bmatrix}$ ✓

d.) $\begin{bmatrix}-2&2\\1/2&2\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}0\\5/2\end{bmatrix} \Rightarrow 0\begin{bmatrix}-1\\2\end{bmatrix}+5/2\begin{bmatrix}2\\0\end{bmatrix}=\begin{bmatrix}5\\0\end{bmatrix}$ ✓ $\boxed{L\left(\begin{bmatrix}1\\2\end{bmatrix}\right)=\begin{bmatrix}5\\0\end{bmatrix}}$

9. $L(1)=0$, $[L(1)]_S=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$, $L(t)=1$, $[L(t)]_S=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$

$L(e^t)=e^t$, $[L(e^t)]_S=\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$, $L(te^t)=e^t+te^t$, $[L(te^t)]_S=\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$

$$\boxed{\begin{bmatrix}0&1&0&0\\0&0&0&0\\0&0&1&1\\0&0&0&1\end{bmatrix}}$$

12. Let $T=\{v_1, v_2, \ldots, v_m\}$ be an ordered basis for U and $S=\{v_1, v_2, \ldots, v_m, v_{m+1}, \ldots, v_n\}$ an ordered basis for V (Theorem 3.10). Now $L(v_j)$ for $j=1,2,\ldots,m$ is a vector in U, so $L(v_j)$ is a linear combination of $v_1, v_2, \ldots, v_m$. Thus $L(v_j)=a_1v_1+a_2v_2+\ldots+a_mv_m+0v_{m+1}+\ldots+0v_n$

Hence $[L(v_j)]_S=\begin{bmatrix}a_1\\a_2\\\vdots\\a_m\\0\\\vdots\\0\end{bmatrix}$. So L with respects to S is of the form $\begin{bmatrix}A&B\\O&C\end{bmatrix}$ where

A is $m\times m$, B is $m\times(n-m)$,
O is $(n-m)\times m$, C is $(n-m)\times(n-m)$

22. a.) $\boxed{[L(v_1)]_T=\begin{bmatrix}1\\-1\end{bmatrix},\ [L(v_2)]_T=\begin{bmatrix}2\\1\end{bmatrix},\ [L(v_3)]_T=\begin{bmatrix}1\\0\end{bmatrix}}$

(columns of representing matrix)

b.) $L(v_1)=1\begin{bmatrix}1\\2\end{bmatrix}-1\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}$

$L(v_2)=2\begin{bmatrix}1\\2\end{bmatrix}+1\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}3\\3\end{bmatrix}$

$L(v_3)=1\begin{bmatrix}1\\2\end{bmatrix}+0\begin{bmatrix}1\\-1\end{bmatrix}=\begin{bmatrix}1\\2\end{bmatrix}$

$$\boxed{L(v_1)=\begin{bmatrix}0\\3\end{bmatrix},\ L(v_2)=\begin{bmatrix}3\\3\end{bmatrix},\ L(v_3)=\begin{bmatrix}1\\2\end{bmatrix}}$$

c) $\left[\begin{bmatrix}2\\1\\-1\end{bmatrix}\right]_S = \begin{bmatrix}2\\-1\\4\end{bmatrix}$, $\begin{bmatrix}2\\1\\-1\end{bmatrix} = 2\begin{bmatrix}-1\\1\\0\end{bmatrix} - 1\begin{bmatrix}0\\1\\1\end{bmatrix} + 4\begin{bmatrix}1\\0\\0\end{bmatrix}$

$$\begin{bmatrix}1 & 2 & 1\\-1 & 1 & 0\end{bmatrix}\begin{bmatrix}2\\-1\\4\end{bmatrix} = \begin{bmatrix}4\\-3\end{bmatrix}$$

$$4\begin{bmatrix}1\\2\end{bmatrix} - 3\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1\\11\end{bmatrix}$$

$$\boxed{L\left(\begin{bmatrix}2\\1\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\11\end{bmatrix}}$$

5.5 5. a) $L\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\0\end{bmatrix}$, $L\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}0\\-1\end{bmatrix}$ $\quad\boxed{\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}}$

b) $L\left(\begin{bmatrix}1\\-1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix} = a\begin{bmatrix}1\\-1\end{bmatrix} + b\begin{bmatrix}1\\2\end{bmatrix} = 1/3\begin{bmatrix}1\\-1\end{bmatrix} + 2/3\begin{bmatrix}1\\2\end{bmatrix}$

$$\left[\begin{array}{cc|c}1 & 1 & 1\\-1 & 2 & 1\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1 & 1\\0 & 3 & 2\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1 & 1\\0 & 1 & 2/3\end{array}\right] \to \left[\begin{array}{cc|c}1 & 0 & 1/3\\0 & 1 & 2/3\end{array}\right]$$

$$\left[L\left(\begin{bmatrix}1\\-1\end{bmatrix}\right)\right]_T = \begin{bmatrix}1/3\\2/3\end{bmatrix}$$

$L\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}1\\-2\end{bmatrix} = a\begin{bmatrix}1\\-1\end{bmatrix} + b\begin{bmatrix}1\\2\end{bmatrix} = 4/3\begin{bmatrix}1\\-1\end{bmatrix} - 1/3\begin{bmatrix}1\\2\end{bmatrix}$

$$\left[\begin{array}{cc|c}1 & 1 & 1\\-1 & 2 & -2\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1 & 1\\0 & 3 & -1\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1 & 1\\0 & 1 & -1/3\end{array}\right] \to \left[\begin{array}{cc|c}1 & 0 & 4/3\\0 & 1 & -1/3\end{array}\right]$$

$$\left[L\left(\begin{bmatrix}1\\2\end{bmatrix}\right)\right]_T = \begin{bmatrix}4/3\\-1/3\end{bmatrix}$$

$$\boxed{\begin{bmatrix}1/3 & 4/3\\2/3 & -1/3\end{bmatrix}}$$

c) $\begin{bmatrix}x\\y\end{bmatrix} = a\begin{bmatrix}1\\-1\end{bmatrix} + b\begin{bmatrix}1\\2\end{bmatrix}$ $\quad \left[\begin{array}{cc|c}1 & 1 & x\\-1 & 2 & y\end{array}\right] \to \left[\begin{array}{cc|c}1 & 1 & x\\0 & 3 & x+y\end{array}\right] \to \left[\begin{array}{cc|c}1 & 0 & 2/3x - 1/3y\\0 & 1 & 1/3x + 1/3y\end{array}\right]$

$$\begin{bmatrix}2/3x - 1/3y\\1/3x + 1/3y\end{bmatrix} = P^{-1}\begin{bmatrix}x\\y\end{bmatrix} \Rightarrow P^{-1} = \begin{bmatrix}2/3 & -1/3\\1/3 & 1/3\end{bmatrix}$$

$$\left[\begin{array}{cc|cc}2/3 & -1/3 & 1 & 0\\1/3 & 1/3 & 0 & 1\end{array}\right] \to \left[\begin{array}{cc|cc}2 & -1 & 3 & 0\\1 & 1 & 0 & 3\end{array}\right] \to \left[\begin{array}{cc|cc}1 & 1 & 0 & 3\\2 & -1 & 3 & 0\end{array}\right] \to \left[\begin{array}{cc|cc}1 & 1 & 0 & 3\\0 & -3 & 3 & -6\end{array}\right]$$

$$\to \left[\begin{array}{cc|cc}1 & 1 & 0 & 3\\0 & 1 & -1 & 2\end{array}\right] \to \left[\begin{array}{cc|cc}1 & 0 & 1 & 1\\0 & 1 & -1 & 2\end{array}\right] \Rightarrow P = \begin{bmatrix}1 & 1\\-1 & 2\end{bmatrix}$$

$$\underset{P^{-1}}{\begin{bmatrix}2/3 & -1/3\\1/3 & 1/3\end{bmatrix}}\underset{A}{\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}}\underset{P}{\begin{bmatrix}1 & 1\\-1 & 2\end{bmatrix}} = \underset{P^{-1}}{\begin{bmatrix}2/3 & -1/3\\1/3 & 1/3\end{bmatrix}}\underset{AP}{\begin{bmatrix}1 & 1\\1 & -2\end{bmatrix}} = \underset{B}{\begin{bmatrix}1/3 & 4/3\\2/3 & -1/3\end{bmatrix}}$$

$\boxed{\text{Yes, the matrices from parts a) and b) are similar}}$

d) $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ has rank 2

$$\begin{bmatrix}1/3 & 4/3\\2/3 & -1/3\end{bmatrix} \to \begin{bmatrix}1 & 4\\2 & -1\end{bmatrix} \to \begin{bmatrix}1 & 4\\0 & -9\end{bmatrix} \to \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \quad \text{rank } 2$$

$\boxed{\text{The rank of each matrix is 2}}$

8. If $B = P^{-1}AP$, the $Tr(B) = Tr(P^{-1}AP) = Tr(APP^{-1})$
$= T(AI_n) = Tr(A)$

$\boxed{\therefore \text{ If } A \text{ and } B \text{ are similar } Tr(B) = Tr(A)}$

11. a) If $B = P^{-1}AP$ and A is nonsingular, then P, P^{-1}, and A can be written as products of elementary matrices. Thus B is a product of elementary matrices and is nonsingular.

b) $B = P^{-1}AP$

$PB = PP^{-1}AP = I_n AP = AP$

$A^{-1}PB = A^{-1}AP = I_n P = P$

$\underbrace{P^{-1}A^{-1}P}_{B^{-1}}B = P^{-1}P = I_n$

$B^{-1} = P^{-1}A^{-1}P$ $\boxed{\therefore B^{-1} \text{ and } A^{-1} \text{ are similar}}$