

Homework #10 Solutions

6.1 1. a) $5 \overbrace{21} 34$ 5 inversions

b) $4 \overbrace{521} 3$ 7 inversions

c) $4 \overbrace{21} 35$ 4 inversions

4. a) 3214 3 inversions \Rightarrow odd

b) 1423 2 inversions \Rightarrow even

c) 2143 2 inversions \Rightarrow even

8. a) $\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 2(2) - 3(-1) = 7$ $\begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 7$

b) $\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 2(3) - 4(1) = 2$ $\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 2$

12. a) $\begin{vmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 2(-3)(4) = -24$ $\begin{vmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = -24$

b) $\begin{vmatrix} 2 & 4 & 5 \\ 0 & -6 & 2 \\ 0 & 0 & 3 \end{vmatrix} = 2(-6)(3) = -36$ $\begin{vmatrix} 2 & 4 & 5 \\ 0 & -6 & 2 \\ 0 & 0 & 3 \end{vmatrix} = -36$

15. a) $\begin{vmatrix} t-1 & 2 \\ 3 & t-2 \end{vmatrix} = (t-1)(t-2) - 6 = t^2 - 3t + 2 - 6$
 $= t^2 - 3t - 4 = (t-4)(t+1) = 0$ $t = -1$ or 4

b) $\begin{vmatrix} t-1 & -1 & -2 \\ 0 & t & 2 \\ 0 & 0 & t-3 \end{vmatrix} = (t-1)t(t-3) = 0$ $t = 0, 1,$ or 3

6.2 1. a) $\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} = 3(1) = 3$ $\begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} = 3$

$$b_1) \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2(1) = 2$$

$$\boxed{\begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} = 2}$$

$$c_1) \begin{vmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 4(2)(3) = 24$$

$$\boxed{\begin{vmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 24}$$

$$d_1) \begin{vmatrix} 4 & 1 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 2 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & 2 \\ 2 & 3 & 0 \\ 4 & 1 & 3 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & 2 \\ 0 & -3 & -4 \\ 0 & -11 & -5 \end{vmatrix} = - \begin{vmatrix} 1 & 3 & 2 \\ 0 & -3 & -4 \\ 0 & 0 & 29/3 \end{vmatrix}$$

$$= - (1)(-3)(29/3) = 29$$

$$\boxed{\begin{vmatrix} 4 & 1 & 3 \\ 2 & 3 & 0 \\ 1 & 3 & 2 \end{vmatrix} = 29}$$

$$e_1) \begin{vmatrix} 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = -1(-1) \begin{vmatrix} 2 & 0 & 0 & 0 \\ 4 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 2(2)(1)(1) = 4$$

$$\boxed{\begin{vmatrix} 4 & 2 & 2 & 0 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{vmatrix} = 4}$$

$$f_1) \begin{vmatrix} 4 & 2 & 3 & -4 \\ 3 & -2 & 1 & 5 \\ -2 & 0 & 1 & -3 \\ 8 & -2 & 6 & 4 \end{vmatrix} = - \begin{vmatrix} -2 & 0 & 1 & -3 \\ 3 & -2 & 1 & 5 \\ 4 & 2 & 3 & -4 \\ 8 & -2 & 6 & 4 \end{vmatrix} = - \begin{vmatrix} -2 & 0 & 1 & -3 \\ 0 & -2 & 5/2 & 1/2 \\ 0 & 2 & 5 & -10 \\ 0 & -2 & 10 & -8 \end{vmatrix}$$

$$= - \begin{vmatrix} -2 & 0 & 1 & -3 \\ 0 & -2 & 5/2 & 1/2 \\ 0 & 0 & 15/2 & -19/2 \\ 0 & 0 & 15/2 & -19/2 \end{vmatrix} = \begin{vmatrix} -2 & 0 & 1 & -3 \\ 0 & -2 & 5/2 & 1/2 \\ 0 & 0 & 15/2 & -19/2 \\ 0 & 0 & 0 & 1 \end{vmatrix} = (-2)(-2)(15/2) = 30$$

$$\boxed{\begin{vmatrix} 4 & 2 & 3 & -4 \\ 3 & -2 & 1 & 5 \\ -2 & 0 & 1 & -3 \\ 8 & -2 & 6 & 4 \end{vmatrix} = 30}$$

$$3. \begin{vmatrix} a_1 + 2b_1 - 3c_1 & a_2 + 2b_2 - 3c_2 & a_3 + 2b_3 - 3c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 3$$

$$\boxed{\det = 3}$$

$$b. b.) \det(A) = \det \begin{pmatrix} 2 & 3 & 6 \\ 0 & 3 & 2 \\ 0 & 0 & -4 \end{pmatrix} = 2(3)(-4) = -24$$

$$\det(B) = \det \begin{pmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ 2 & 1 & -2 \end{pmatrix} = 3(5)(-2) = -30$$

$$\det(A)\det(B) = 720$$

$$\det(AB) = \det \left(\begin{pmatrix} 2 & 3 & 6 \\ 0 & 3 & 2 \\ 0 & 0 & -4 \end{pmatrix} \begin{pmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ 2 & 1 & -2 \end{pmatrix} \right) = \det \begin{pmatrix} 30 & 21 & -12 \\ 16 & 17 & -4 \\ -8 & -4 & 8 \end{pmatrix}$$

$$= -\det \begin{pmatrix} -8 & -4 & 8 \\ 16 & 17 & -4 \\ 30 & 21 & -12 \end{pmatrix} = -\det \begin{pmatrix} -8 & -4 & 8 \\ 0 & 9 & 12 \\ 0 & 6 & 18 \end{pmatrix}$$

$$= -\det \begin{pmatrix} -8 & -4 & 8 \\ 0 & 9 & 12 \\ 0 & 0 & 10 \end{pmatrix} = -(-8)(9)(10) = 720$$

$$\det(AB) = 720$$

$$\therefore \det(A)\det(B) = \det(AB)$$

