Homework #10 Solutions

6.1  
a) 5 2 1 3 4  \hspace{1cm} 5 \text{ inversions}  
b) 4 5 2 1 3  \hspace{1cm} 7 \text{ inversions}  
c) 4 2 1 3 5  \hspace{1cm} 4 \text{ inversions}  

4. a) 3 2 1 4  \hspace{1cm} 3 \text{ inversions} \Rightarrow \text{ odd}  
b) 1 4 2 3  \hspace{1cm} 2 \text{ inversions} \Rightarrow \text{ even}  
c) 2 1 4 3  \hspace{1cm} 2 \text{ inversions} \Rightarrow \text{ even}  

8. a) 2 1 -1  \hspace{1cm} 2(2) - 3(-1) = 7  
     \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix} = 7  
b) 1 3 2  \hspace{1cm} 2(3) - 4(1) = 2  
     \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} = 2  

12. a) 2 0 0 0  \hspace{1cm} 2(-3)(4) = -24  
     \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 4 \end{bmatrix} = -24  
b) 2 4 5  \hspace{1cm} 2(-6)(3) = -36  
     \begin{bmatrix} 2 & 4 & 5 \\ 0 & -6 & 2 \\ 0 & 0 & 3 \end{bmatrix} = -36  

15. a) \begin{vmatrix} t-1 & 2 \\ \frac{1}{3} & t-2 \end{vmatrix} = (t-1)(t-2) - 6 = t^2 - 3t + 2 - 6  
     \Rightarrow t^2 - 3t - 4 = (t-4)(t+1) = 0  \hspace{1cm} t = -1 \text{ or } 4  
b) \begin{vmatrix} t-1 & -1 \\ \frac{1}{3} & t-2 \end{vmatrix} = (t-1)t(t-3) = 0  \hspace{1cm} t = 0, 1, \text{ or } 3  

6.2  
a) \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} = 3(1) = 3  
     \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} = 3
b) \[
\begin{vmatrix}
1 & 4 & 0 \\
3 & 0 & 2 \\
1 & 0 & 3
\end{vmatrix} = 2 \]
\[
\begin{vmatrix}
2 & 1 \\
0 & 1
\end{vmatrix} = 2
\]
\[
| 2_1 4_3 | = 2
\]

c) \[
\begin{vmatrix}
0 & 0 & 4 \\
2 & 0 & 0 \\
0 & 0 & 3
\end{vmatrix} = 24
\]
\[
\begin{vmatrix}
4 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 3
\end{vmatrix} = 24
\]
\[
| 4_0 0_0 | = 4(2)(3) = 24
\]

\[
\begin{vmatrix}
1 & 3 & 2 \\
3 & 0 & 0 \\
1 & 3 & 2
\end{vmatrix} = -132
\]
\[
\begin{vmatrix}
1 & 3 & 2 \\
0 & -3 & -4 \\
0 & 1 & -5
\end{vmatrix} = -132
\]
\[
= -(1)(-3)(29/3) = 29
\]
\[
\begin{vmatrix}
1 & 3 \\
2 & 3 \\
1 & 3
\end{vmatrix} = 29
\]

\[
\begin{vmatrix}
4 & 2 & 0 \\
2 & 0 & 0 \\
0 & 0 & 1
\end{vmatrix} = -1
\]
\[
\begin{vmatrix}
2 & 0 & 0 \\
4 & 2 & 0 \\
0 & 0 & 1
\end{vmatrix} = 2
\]
\[
\begin{vmatrix}
2 & 0 & 0 \\
0 & 2 & 0 \\
0 & 0 & 1
\end{vmatrix} = 2(2)(1)(1) = 1
\]

\[
\begin{vmatrix}
4 & 2 & 0 \\
2 & 0 & 0 \\
0 & 0 & 1
\end{vmatrix} = 4
\]

\[
\begin{vmatrix}
4 & 2 & 3 & -4 \\
3 & -2 & 1 & 5 \\
-2 & 0 & -1 & 3 \\
8 & -2 & 0 & 6
\end{vmatrix} = -2013
\]
\[
\begin{vmatrix}
3 & -2 & 1 & 5 \\
4 & 2 & 3 & -4 \\
8 & -2 & 0 & 6
\end{vmatrix} = -2013
\]
\[
\begin{vmatrix}
-2 & 0 & 1 & -3 \\
0 & -2 & 5/2 & 1/2 \\
0 & -2 & 0 & 15/2 \\
0 & 0 & 15/2 & -19/2
\end{vmatrix} = (-2)(-2)(15/2)
\]
\[
= 30
\]

\[
\begin{vmatrix}
4 & 2 & 3 & -4 \\
3 & -2 & 1 & 5 \\
-2 & 0 & 1 & -3 \\
8 & -2 & 0 & 6
\end{vmatrix} = 30
\]

3. \[
\begin{vmatrix}
a_1 & +2b_1 & -3c_1 \\
a_2 & +2b_2 & -3c_2 \\
a_3 & +2b_3 & -3c_3
\end{vmatrix}
\]
\[
\begin{vmatrix}
b_1 \\
b_2 \\
b_3
\end{vmatrix}
\]
\[
\begin{vmatrix}
c_1 \\
c_2 \\
c_3
\end{vmatrix}
\]
\[
= \begin{vmatrix}
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
c_1 & c_2 & c_3
\end{vmatrix} = 3
\]
\[
\text{det} = 3
\]
(b) \( \det(A) = \det \begin{bmatrix} 2 & 3 & 6 \\ 0 & -3 & 2 \\ 0 & 0 & -4 \end{bmatrix} = 2(3)(-4) = -24 \)

\( \det(B) = \det \begin{bmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ 2 & 1 & -2 \end{bmatrix} = 3(5)(-2) = -30 \)

\[ \det(A) \det(B) = 720 \]

\[ \det(AB) = \det \begin{bmatrix} 2 & 3 & 6 \\ 0 & 3 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 4 & 5 & 0 \\ 2 & 1 & -2 \end{bmatrix} = \det \begin{bmatrix} 30 & 21 & -12 \\ 16 & 17 & -4 \\ -8 & -4 & 8 \end{bmatrix} \]

= \(-\det \begin{bmatrix} -8 & -4 & 8 \\ 16 & 17 & -4 \\ 30 & 21 & -12 \end{bmatrix}\) = \(-\det \begin{bmatrix} -8 & -4 & 8 \\ 0 & 9 & 12 \\ 0 & 6 & 18 \end{bmatrix}\)

= \(-\det \begin{bmatrix} -8 & -4 & 8 \\ 0 & 9 & 12 \\ 0 & 0 & 10 \end{bmatrix}\) = \(-8)(7)(10) = 720 \]

\[ \det(AB) = 720 \]

\[ \therefore \quad \det(A) \det(B) = \det(AB) \]