Ma 309 — Matrix Algebra
Practice Exam

Prof. Sawyer — Washington Univ. — May 2, 2007

Calculators cannot be used. You can use both sides of a one-page crib sheet.
Take 1 hour and 15 minutes (7 problems).

1. Find the dimension of the span of the following four vectors in R^4:

$$\begin{aligned} v_1 &= [\ 1 \quad 3 \quad 4 \quad 0] \\ v_2 &= [\ 0 \quad 5 \quad 2 \quad 0] \\ v_3 &= [-1 \quad 7 \quad 0 \quad 0] \\ v_r &= [\ 2 \quad 1 \quad 1 \quad 5] \end{aligned}$$

2. Find a basis for the null space of the matrix

$$\begin{bmatrix} 1 & 5 & 3 & 4 \\ 0 & 1 & 3 & 1 \\ 1 & 4 & 0 & 3 \end{bmatrix}$$

3. Can the 2×2 identity matrix I_2 be written as a linear combination of the matrices $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$? Prove or disprove.

4. Find the set of all vectors x in R^3 such that

$$\begin{bmatrix} 5 & 4 & 7 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

Is the set of solutions a vector subspace of R^3? (True or false.)

5. Find a basis for the orthogonal complement in R^5 of the two vectors

$$\begin{aligned} v_1 &= [1 \quad 5 \quad 4 \quad 5 \quad 3] \qquad \text{and} \\ v_2 &= [1 \quad 3 \quad 0 \quad 3 \quad 1] \end{aligned}$$

6. Find the orthogonal projection of $v = [2 \quad 1 \quad 0]$ onto the line through the vector $u = [0 \quad 2 \quad 1]$. Show that the projection must be of the form cu for some constant c and that, indeed, $v - cu$ and cu are orthogonal.

7. Find the characteristic polynomial and the eigenvalues of the matrix

$$\begin{bmatrix} 2 & -2 & 3 \\ 0 & 3 & -2 \\ 0 & -1 & 2 \end{bmatrix}$$