Ch. 2 #6, 9, 10, 11, 12, 14, 16, 17, 18

Answer Key

2.6 Out of the 54 numbers, 6 of them will be chosen for the lottery and 48 will not. So for the grand prize, the probability of winning is

$$\frac{\binom{6}{6}\binom{48}{0}}{\binom{54}{6}} = \frac{1}{25{,}827{,}165} = 3.9 \times 10^{-8}$$

For the 2nd prize, the probability of winning is

$$\frac{\binom{6}{5}\binom{48}{1}}{\binom{54}{6}} = \frac{288}{25{,}827{,}165} = 1.12 \times 10^{-5}$$

For the third prize, the probability of winning is

$$\frac{\binom{6}{4}\binom{48}{2}}{\binom{54}{6}} = \frac{16{,}920}{25{,}827{,}165} = 6.55 \times 10^{-4}$$

2.9

$$\binom{n-1}{r-1} + \binom{n-1}{r} = \frac{(n-1)!}{(r-1)!\,(n-r)!} + \frac{(n-1)!}{r!\,(n-r)!}$$

$$= \frac{n!}{r!\,(n-r)!}\left(\frac{r}{n} + \frac{n-r}{n}\right)$$

$$= \binom{n}{r}$$

n	Coefficients
0	1
1	1 1
2	1 2 1
3	1 3 3 1
4	1 4 6 4 1
5	1 5 10 10 5 1

2.10 a. $\binom{7}{2} = 21$

b. $\binom{7}{2}(2)^2(-3)^5 = -20,412$

c. $\frac{7!}{2!\,2!\,3!} = 210$

2.11 a. $$\frac{\binom{12}{3}}{\binom{34}{3}} = \frac{220}{5984} = .0368$$

b. $$\frac{\binom{2}{2}\binom{32}{1}}{\binom{34}{3}} = \frac{32}{5984} = .005$$

c. $$\frac{\binom{2}{2}\binom{32}{1}}{\binom{34}{3}} + \frac{\binom{12}{2}\binom{22}{1}}{\binom{34}{3}} + \frac{\binom{20}{2}\binom{14}{1}}{\binom{34}{3}} =$$

$$\frac{4144}{5984} = .693$$

d. $$\frac{\binom{2}{2}\binom{32}{1}}{\binom{34}{3}} + \frac{\binom{12}{2}\binom{22}{1}}{\binom{34}{3}} + \frac{\binom{20}{2}\binom{14}{1}}{\binom{34}{3}} + \frac{\binom{12}{3}\binom{22}{0}}{\binom{34}{3}}$$

$$+ \frac{\binom{20}{3}\binom{14}{0}}{\binom{34}{3}} = \frac{5504}{5984} = .920$$

2.12 a. $\frac{25}{30} \times \frac{5}{29} = 0.144$

b. $\frac{25}{30} \times \frac{5}{29} + \frac{5}{30} \times \frac{4}{29} = 0.167$

c. $\frac{5}{30} \times \frac{4}{29} \times \frac{3}{28} = 0.002$

2.14 If we order the 12 kids by team, then there are 12! ways to assign the performance ranks. However, within each team, order is irrelevant, so we need to divide out the 3! ways of ordering the 3 kids per team. So the final number of ways of ranking the teams is

$$\frac{12!}{3!3!3!3!} = 369{,}600$$

2.16 $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.6} = 0.5$

$$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{0.3}{0.6} = 0.5$$

$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{0.5 - 0.3}{1 - 0.6} = 0.5$$

$$P(B^c|A \cap C) = \frac{P(A \cap B^c \cap C)}{P(A \cap C)} = \frac{P(A \cap C) - P(A \cap B \cap C)}{P(A \cap C)} = \frac{0.2 - 0.1}{0.2} = 0.5$$

2.17 a. For A and B to be mutually exclusive, $P(A \cap B) = 0$ or $P(A \cup B) = P(A) + P(B)$. Then $0.8 = 0.4 + p$, which means that $p = 0.4$.

b. For A and B to be independent, $P(A \cap B) = P(A)P(B)$. Then $P(A \cap B) =$
$P(A) + P(B) - P(A \cup B) = 0.4 + p - 0.8 =$

$P(A)P(B) = 0.4p$, which means that $p = 2/3$.

2.18 a. $P(N \mid R) = \dfrac{P(N \cap R)}{P(R)} = \dfrac{0.21}{0.47} = 0.447$

b. $P(R^c \mid T) = \dfrac{P(R^c \cap T)}{P(T)} = \dfrac{P(T) - P(R \cap T)}{P(T)} =$

$\dfrac{0.77 - 0.29}{0.77} = 0.623$

c. $P(T^c \mid N \cap R) = \dfrac{P(T^c \cap N \cap R)}{P(N \cap R)} =$

$\dfrac{P(N \cap R) - P(N \cap R \cap T)}{P(N \cap R)} = \dfrac{0.21 - 0.06}{0.21} =$

0.714

Ch. 2 # 20, 22, 27, 28, 29, 30, 32, 33, 34

2.20 Let A_i & P_i denote the events that an ace or a face card is drawn on the ith draw, respectively. Then:

$$P(\text{ace before face card}) = P(A_1) + P((A_1 \cup P_1)^c)P(A_2) + \ldots$$
$$= \frac{4}{52} + \left(\frac{36}{52}\right)\frac{4}{52} + \left(\frac{36}{52}\right)^2\frac{4}{52} + \ldots$$
$$= \frac{4}{52}\sum_{i=0}^{\infty}\left(\frac{36}{52}\right)^i$$
$$= \frac{4}{52}\left(\frac{1}{1-\frac{36}{52}}\right) = 0.25$$

2.22 Place of residence & opinion on a tax increase are not independent, since

$$P(\text{Yes \& City}) = \frac{100}{1000} = 0.1 \neq$$

$$P(\text{Yes})P(\text{City}) = \frac{400}{1000} \times \frac{400}{1000} = 0.16$$

2.27 a Let D & ND refer to the events where a person has or doesn't have the disease, respectively. Then

$$P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|ND)P(ND)} = \frac{0.99 \times 0.1}{0.99 \times 0.1 + 0.02 \times 0.9}$$
$$= 0.846$$

b.

$$P(ND|-) = \frac{P(-|ND)P(ND)}{P(-|ND)P(ND) + P(-|D)P(D)} = \frac{0.98 \times 0.9}{0.98 \times 0.9 + 0.01 \times 0.1}$$
$$= 0.999$$

The diagnostic test appears pretty reliable, although it is less reliable in identifying true positives than true negatives.

.27 ctd.

c. $P(D|+) = \dfrac{P(+|D)P(D)}{P(+|D)P(D) + P(+|ND)P(ND)}$

$= \dfrac{0.99 \times 0.001}{0.99 \times 0.001 + 0.02 \times 0.999} = 0.047$

d. For rare diseases, too many false positives would appear in the screening program, & it would not be very effective in identifying people with the disease.

2.28 a. For this to be a p.m.f.,

$\sum f(x) = c\left(\frac{1}{2}\right) + c\left(\frac{1}{4}\right) + c\left(\frac{1}{8}\right) + c\left(\frac{1}{16}\right) = 1$, or $c = \frac{16}{15} = 1.067$

b. The cdf is

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ 1.067/2 = 0.533 & \text{if } 1 \le x < 2 \\ 0.533 + 1.067/4 = 0.8 & \text{if } 2 \le x < 3 \\ 0.8 + 1.067/8 = 0.933 & \text{if } 3 \le x < 4 \\ 1 & \text{if } x \ge 4 \end{cases}$$

2.29 a.

x	$f(x)$	$F(y)$, for $x \le y < x+1$
0	6/36	6/36
1	10/36	16/36
2	8/36	24/36
3	6/36	30/36
4	4/36	34/36
5	2/36	1

b. $P(0 < x < 3) = F(3) - F(0) = 24/36$

$P(1 \le x < 3) = F(2) - F(0) = 18/36$

2.30 a For this to be a p.m.f.,

$$\sum_x f(x) = \sum_{x=1}^{\infty} \frac{1}{n(n+1)} = 1.$$

Since

$$\sum_{x=1}^{n} \frac{1}{n(n+1)} = \frac{n}{n+1},$$

then

$$\sum_{x=1}^{\infty} = \lim_{n \to \infty} \frac{n}{n+1} = 1,$$

& $f(x)$ is a p.m.f.

b. From (a) the c.d.f. is

$$F(x) = \begin{cases} 0 & \text{if } x < 1 \\ \frac{i}{i+1} & \text{if } i \leq x < i+1 \end{cases}$$

2.32 a For this to be a p.d.f.,

$$\int_x f(x) = \int_0^1 0.5\,dx + \int_1^3 (0.5 + c(x-1))\,dx = 1$$

Since

$$\int_x f(x) = 0.5x\Big|_0^1 + 0.5x\Big|_1^3 + \frac{c}{2}(x-1)^2\Big|_1^3$$
$$= 1.5 + \frac{c}{2}(4-0) = 1.5 + 2c = 1,$$

then c must be -0.25.

b. Using the value of c found above, the new p.d.f. for $1 \leq X < 3$ is $f(x) = 0.5 - 0.25(x-1) = 0.75 - 0.25x$.

The cdf for x between 0 & 1 is

$$F(x) = \int_0^x 0.5\,dx = 0.5x$$

At $x=1$, $F(1) = 0.5$, so the cdf for x between 1 & 3 is

$$F(x) = F(1) + \int_1^x (0.75 - 0.25x)\,dx = 0.5 + (0.75x - 0.125x^2)\Big|_1^x$$
$$= -0.125x^2 + 0.75x - 0.125$$

→

2.32 b ctd Then the final cdf is given by

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.5x & \text{if } 0 \le x < 1 \\ -0.125x^2 + 0.75x - 0.125 & \text{if } 1 \le x < 3 \\ 1 & \text{if } x \ge 3 \end{cases}$$

2.33 a. Continuous

b. $P(1 \le X \le 3) = F(3) - F(1) = 0.8 - 0.4 = 0.4$

c. $P(X \ge 1) = 1 - F(1) = 1 - 0.4 = 0.6$

2.34 a. Discrete

b. $P(1 \le X < 2) = P(X = 1) = 0.8 - 0.4 = 0.4$

c. $P(X \ge 1) = 1 - F(0) = 1 - 0.4 = 0.6$