Chapter 13 Solutions

Solutions to Section 13.1

13.1 (a) Since $\bar{y}_{...} = \frac{4.85+3.72+...+5.34}{6} = 4.637$, the estimated effects are:

Age	Breed			Row
	Guernsey	Holstein-Fresian	Jersey	Effects
Mature	$\widehat{(\tau\beta)}_{11} = -0.067$	$\widehat{(\tau\beta)}_{12} = 0.083$	$\widehat{(\tau\beta)}_{13} = -0.017$	$\hat{\tau}_1 = -0.033$
Young	$\widehat{(\tau\beta)}_{21} = 0.067$	$\widehat{(\tau\beta)}_{22} = -0.083$	$\widehat{(\tau\beta)}_{23} = 0.017$	$\hat{\tau}_2 = 0.033$
Column Effects	$\hat{\beta}_1 = 0.313$	$\hat{\beta}_2 = -0.967$	$\hat{\beta}_3 = 0.653$	

(b) MSE is the weighted average of the within group variances. Since all of the sample sizes are equal,

$$s^2 = \frac{s_1^2 + s_2^2 + \ldots + s_6^2}{6} = \frac{(0.503)^2 + (0.329)^2 + \ldots + (0.674)^2}{6} = 0.227$$

Since there are $10 - 1 = 9$ degrees of freedom per group, this estimate of MSE has $9 \times 6 = 54$ degrees of freedom.

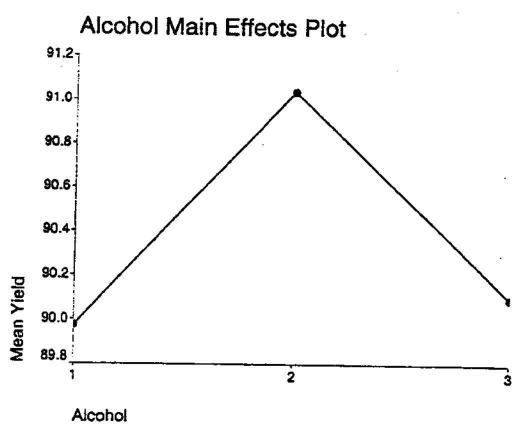
(c)

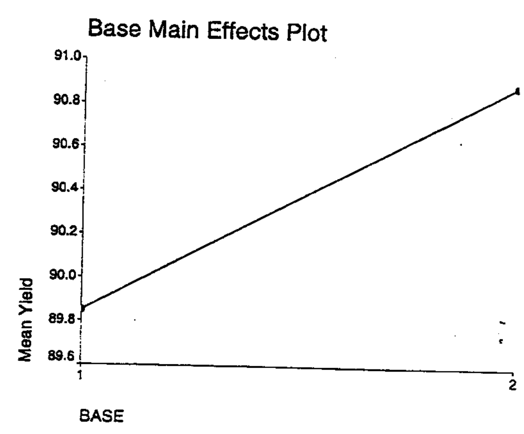
Analysis of Variance

Source	SS	d.f.	MS	F	Sig.
Age	0.067	1	0.067	0.294	0.590
Breed	29.189	2	14.595	64.414	0.000
Interaction	0.233	2	0.117	0.515	0.600
Error	12.235	54	0.227		
Total	41.724	59			

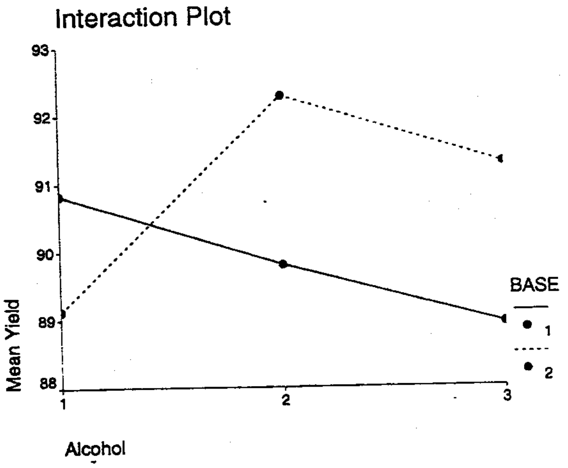
There is no significant interaction effect($P = 0.600$). Breed is the only significant main effect($P = 0.000$).

13.2 (a)





While both plots appear to show "large" main effects, notice that the means only vary between 90 and 91. So the scale is deceiving here, and the effect is probably not as large as it appears.



Since the lines cross substantially, there appears to be a significant interaction effect.

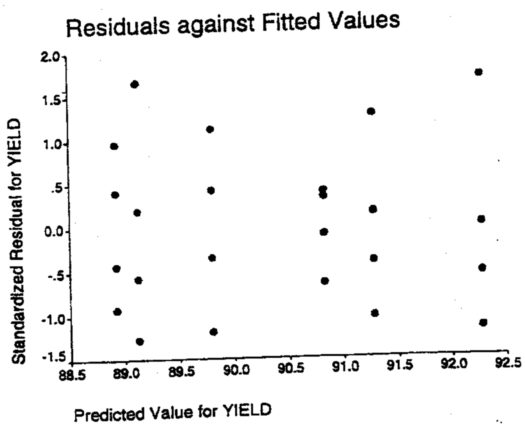
(b)

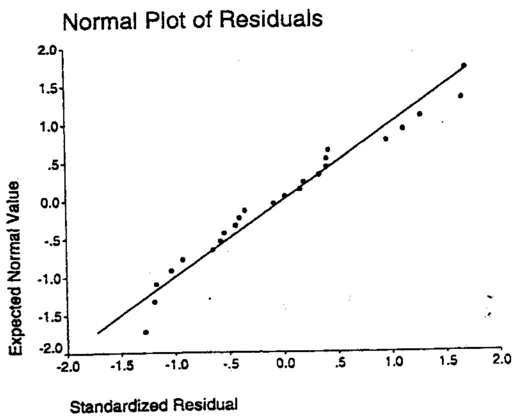
Analysis of Variance

Source	SS	d.f.	MS	F	Sig.
Alcohol	5.396	2	2.698	1.321	0.291
Base	6.510	1	6.510	3.188	0.091
Interaction	22.566	2	11.283	5.525	0.013
Error	36.758	18	2.042		
Total	71.230	23			

Only the Alcohol X Base interaction effect is significant at $\alpha = 0.05$.

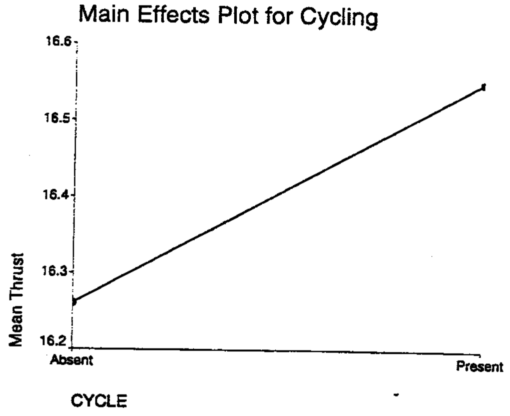
(c)

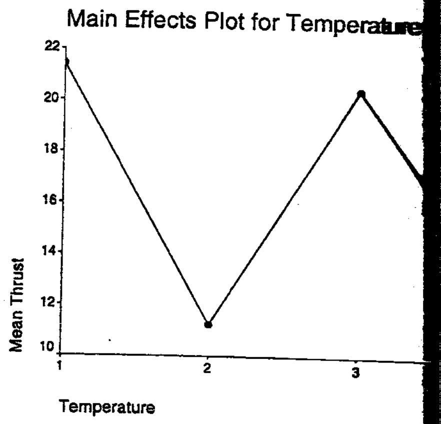




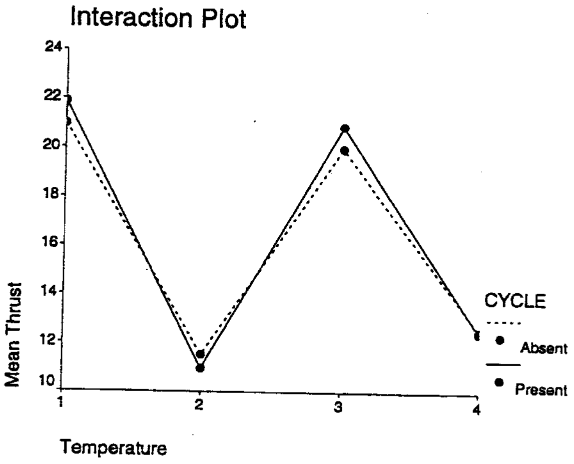
The normal plot is fairly linear, so the normality assumption is valid. The plot of residuals vs. the fitted values is fairly random, so the assumption of constant vari is valid.

13.3 (a)





The means for cycling do not vary much, and so cycling does not have a large main eff However, the means for temperature vary substantially, and this effect is probably la



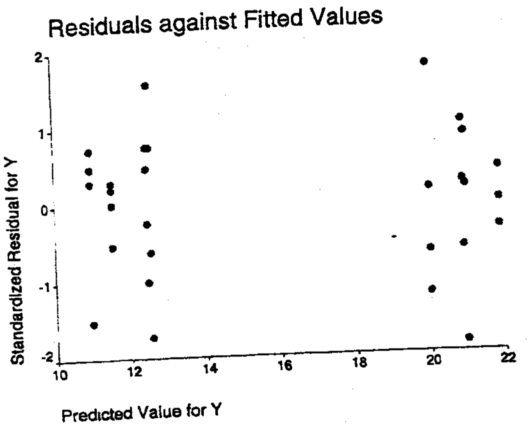
The lines are parallel and almost identical, indicating that the interaction effect is ne ligible.

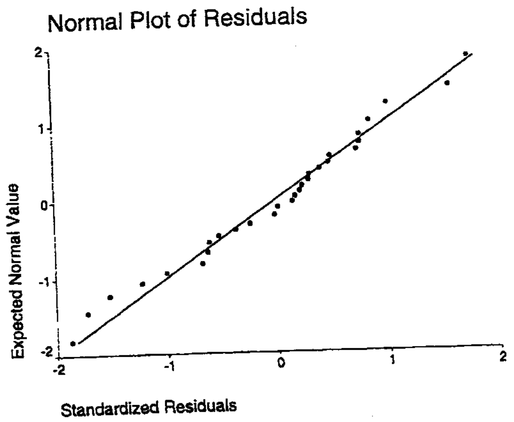
(b)

Analysis of Variance					
Source	SS	d.f.	MS	F	Sig.
Cycling	0.667	1	0.667	1.199	0.284
Temperature	667.132	3	222.377	399.634	0.000
Interaction	3.283	3	1.094	1.967	0.146
Error	13.355	24	0.556		
Total	684.437	31			

There is no significant interaction effect. Temperature is the only significant main effect.

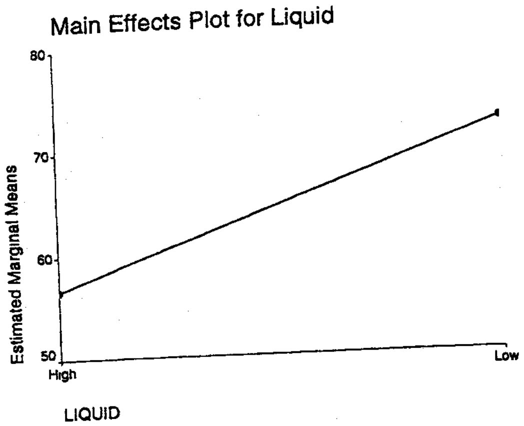
(c)

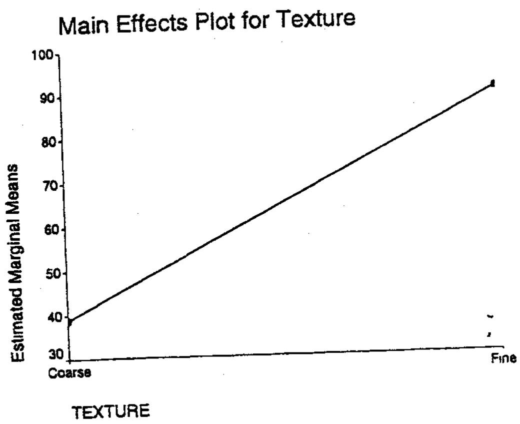




The plot of the residuals against the fitted values appears random and with the same spread, so the equal variance assumption is reasonable. The normal plot looks linear, so the assumption of normality is reasonable.

13.4 (a)





The plot against the fitted values appears random and evenly spread, so the constant variance assumption is valid. The normal plot appears linear, so the normal assumption is reasonable.

13.5 The general form of the Tukey simultaneous CI is

$$\bar{y}_i - \bar{y}_j \pm q_{b,\nu,\alpha} s/\sqrt{an}$$

Using the critical value $q_{3,54,0.05} = 3.41$ and the MSE estimate of variance $s^2 = 0.227$, the margin of error is

$$3.44\sqrt{\frac{0.227}{2 \times 10}} = 0.366.$$

Then the CI's are

$$\begin{aligned}
1 \text{ vs. } 2 &: \; 4.95 - 3.67 \pm 0.366 = [0.914, 1.646] \\
1 \text{ vs. } 3 &: \; 4.95 - 5.29 \pm 0.366 = [-0.706, 0.026] \\
2 \text{ vs. } 3 &: \; 3.67 - 5.29 \pm 0.366 = [-1.986, -1.254]
\end{aligned}$$

Group 2 (Holstein-Fresian) produces significantly lower amounts of butterfat than the other two breeds of cows.

13.6 The general form of the Tukey simultaneous CI is

$$\bar{y}_i - \bar{y}_j \pm q_{b,\nu,\alpha} s/\sqrt{an}$$

Using the critical value $q_{4,24,0.05} = 3.90$ and the MSE estimate of variance $s^2 = 0.556$, the margin of error is

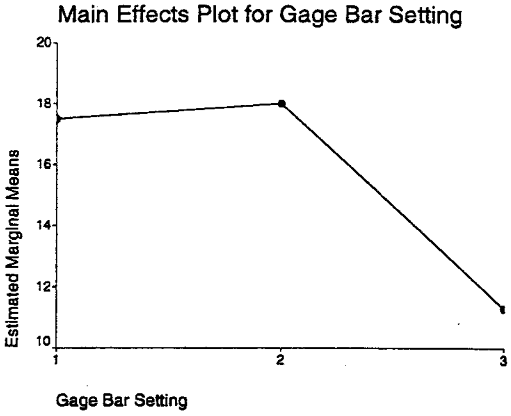
$$3.90\sqrt{\frac{0.556}{2 \times 4}} = 1.028.$$

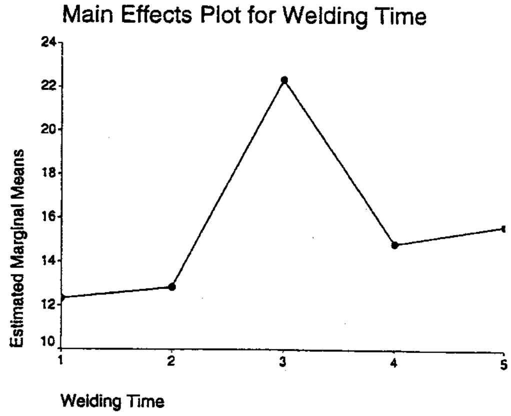
Then the CI's are

$$\begin{aligned}
1 \text{ vs. } 2 &: \; 21.419 - 11.243 \pm 1.028 = [9.148, 11.204] \\
1 \text{ vs. } 3 &: \; 21.419 - 20.454 \pm 1.028 = [-0.067, 1.993] \\
1 \text{ vs. } 4 &: \; 21.419 - 12.505 \pm 1.028 = [7.882, 9.942] \\
2 \text{ vs. } 3 &: \; 11.243 - 20.454 \pm 1.028 = [-10.248, -8.183] \\
2 \text{ vs. } 4 &: \; 11.243 - 12.505 \pm 1.028 = [-2.290, -0.234] \\
3 \text{ vs. } 4 &: \; 20.454 - 12.505 \pm 1.028 = [6.921, 8.977]
\end{aligned}$$

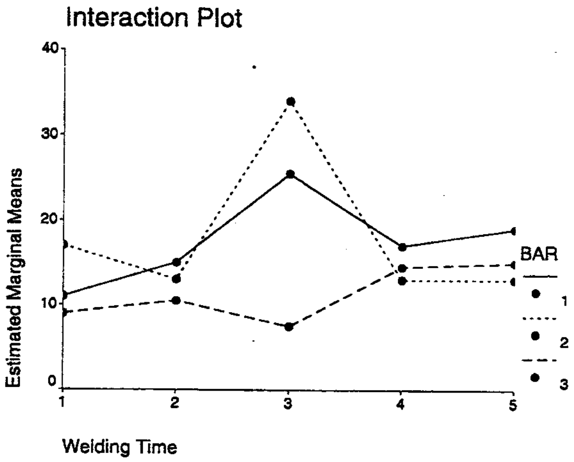
Temperatures 1 and 3 have significantly higher rocket thrust than temperatures 2 and 4. Temperature 2 has significantly lower thrust than temperature 4.

13.7 (a)





Both factors appear to have a sizeable effect on the means.



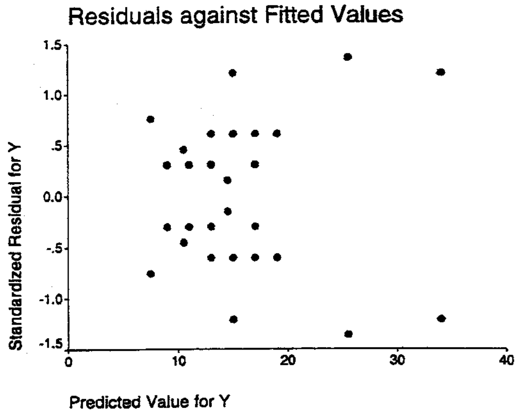
The lines are parallel except for time 3, where they all move in different directions and cross. This suggests that there is a significant interaction effect.

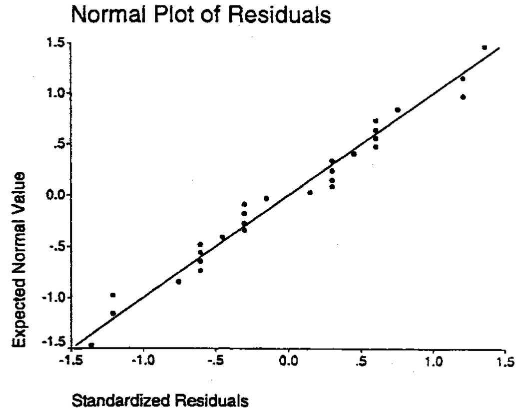
(b)

Analysis of Variance					
Source	SS	d.f.	MS	F	Sig.
Bar	278.600	2	139.300	12.741	0.001
Time	385.533	4	96.383	8.816	0.001
Interaction	597.067	8	74.633	6.826	0.001
Error	164.000	15	10.933		
Total	1425.200	29			

The interaction effect is significant. For bar settings 1 and 2, the time of greatest weld strength is 3, while for bar setting 3, the time of greatest weld strength is 4 or 5.

(c)





The plot against the fitted values appears random and evenly spread, so the assumption of constant variance is valid. The normal plot appears linear, so the assumption of normality is reasonable.

13.8 The regression output is below

```
Regression Analysis


The regression equation is
Y = 15.6 - 3.27 v1 - 2.77 v2 + 6.73 v3 - 0.77 v4 + 1.90 u1 + 2.40 u2
           - 3.23 u1v1 + 2.27 u2v1 + 0.27 u1v2 - 2.23 u2v2 + 1.27 u1v3
           + 9.27 u2v3 + 0.27 u1v4 - 4.23 u2v4
```

Predictor	Coef	StDev	T	P
Constant	15.6000	0.6037	25.84	0.000
v1	-3.267	1.207	-2.71	0.016
v2	-2.767	1.207	-2.29	0.037
v3	6.733	1.207	5.58	0.000
v4	-0.767	1.207	-0.63	0.535
u1	1.9000	0.8537	2.23	0.042
u2	2.4000	0.8537	2.81	0.013
u1v1	-3.233	1.707	-1.89	0.078
u2v1	2.267	1.707	1.33	0.204
u1v2	0.267	1.707	0.16	0.878
u2v2	-2.233	1.707	-1.31	0.211
u1v3	1.267	1.707	0.74	0.470
u2v3	9.267	1.707	5.43	0.000
u1v4	0.267	1.707	0.16	0.878
u2v4	-4.233	1.707	-2.48	0.026

S = 3.307   R-Sq = 88.5%   R-Sq(adj) = 77.8%

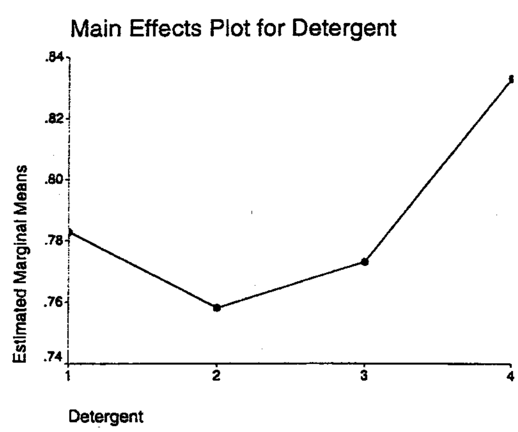
Analysis of Variance

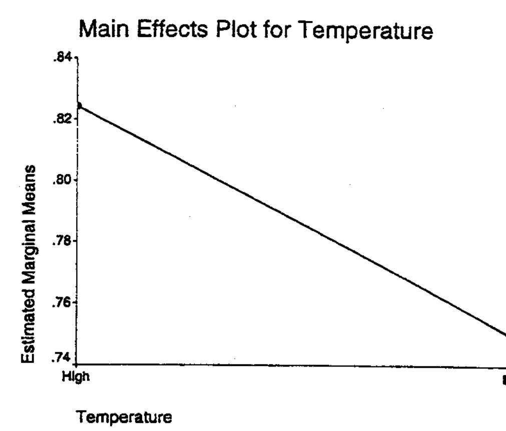
Source	DF	SS	MS	F	P
Regression	14	1261.20	90.09	8.24	0.000
Residual Error	15	164.00	10.93		
Total	29	1425.20			

13.9 (a) The transformed data are:

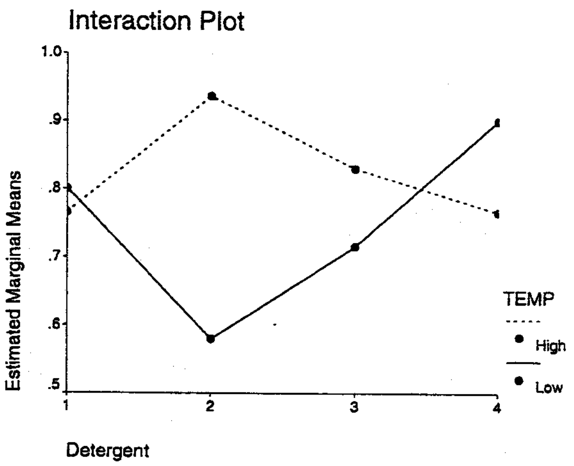
Temperature	Detergent							
	1		2		3		4	
Low	0.766	0.835	0.645	0.515	0.736	0.696	0.885	0.916
High	0.825	0.706	0.947	0.926	0.855	0.805	0.746	0.785

(b)





Both factors appear to have a sizeable effect on the means.



Since the two lines move in opposite directions and cross, the interaction effect appears to be significant.

(c)

Analysis of Variance					
Source	SS	d.f.	MS	F	Sig.
Temperature	0.0226	1	0.0226	8.440	0.020
Detergent	0.0127	3	0.0042	1.571	0.270
Interaction	0.1370	3	0.0456	16.988	0.001
Error	0.0215	8	0.0027		
Total	0.1930	15			

The main effect of temperature and the interaction effect are significant.

13.10 The overall regression equation is

$$Y = \mu + \tau_1 u_1 + \beta_1 v_1 + \beta_2 v_2 + (\tau\beta)_{11} u_1 v_1 + (\tau\beta)_{12} u_1 v_2 + \epsilon.$$

The models for the six treatment combinations are:

$$\begin{aligned}
i = 1, j = 1 &: \; Y = \mu + \tau_1 + \beta_1 + (\tau\beta)_{11} \\
i = 1, j = 2 &: \; Y = \mu + \tau_1 + \beta_2 + (\tau\beta)_{12} \\
i = 1, j = 3 &: \; Y = \mu + \tau_1 - \beta_1 - \beta_2 - (\tau\beta)_{11} - (\tau\beta)_{12} \\
i = 2, j = 1 &: \; Y = \mu - \tau_1 + \beta_1 - (\tau\beta)_{11} \\
i = 2, j = 2 &: \; Y = \mu - \tau_1 + \beta_2 - (\tau\beta)_{12} \\
i = 2, j = 3 &: \; Y = \mu - \tau_1 - \beta_1 - \beta_2 + (\tau\beta)_{11} + (\tau\beta)_{12}
\end{aligned}$$

13.11 The regression output is shown below

```
Regression Analysis
```

Source	DF	SS	MS	F	P
Regression	7	0.172030	0.024576	9.17	0.003
Residual Error	8	0.021436	0.002680		
Total	15	0.193466			

13.13 (a)

	Analysis of Variance				
Source	SS	d.f.	MS	F	Sig.
Biological	2275.8	1	2275.8	13.02	0.001
Adoptive	1277.4	1	1277.4	7.31	0.011
Biol*Adopt	1.9	1	1.9	0.01	0.917
Error	5941.2	34	174.7		
Total	9712.2	37			

The interaction effect is nonsignificant. All main effects are significant.

(b) The regression outputs are

```
Regression Analysis for Full Model


The regression equation is
IQ = 106 - 7.77 u1 - 5.83 v1 + 0.23 u1v1

Predictor        Coef       StDev        T         P
Constant      105.775       2.154    49.10     0.000
u1             -7.775       2.154    -3.61     0.001
v1             -5.825       2.154    -2.70     0.011
u1v1            0.225       2.154     0.10     0.917

S = 13.22        R-Sq = 38.8%      R-Sq(adj) = 33.4%

Analysis of Variance

Source           DF          SS          MS        F        P
Regression        3      3771.0      1257.0     7.19    0.001
Residual Error   34      5941.2       174.7
Total            37      9712.2


Regression Analysis for Partial Model without Interaction


The regression equation is
IQ = 106 - 7.79 u1 - 5.81 v1

Predictor        Coef       StDev        T         P
Constant      105.788       2.120    49.90     0.000
```

u1	-7.788	2.120	-3.67	0.001
v1	-5.812	2.120	-2.74	0.010

S = 13.03 R-Sq = 38.8% R-Sq(adj) = 35.3%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	3769.1	1884.6	11.10	0.000
Residual Error	35	5943.1	169.8		
Total	37	9712.2			

Regression Analysis for Partial Model without Adoptive

The regression equation is
IQ = 106 - 8.12 u1 - 0.12 u1v1

Predictor	Coef	StDev	T	P
Constant	106.118	2.337	45.42	0.000
u1	-8.118	2.337	-3.47	0.001
u1v1	-0.118	2.337	-0.05	0.960

S = 14.36 R-Sq = 25.7% R-Sq(adj) = 21.4%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	2493.6	1246.8	6.05	0.006
Residual Error	35	7218.6	206.2		
Total	37	9712.2			

Regression Analysis for Partial Model without Biological

The regression equation is
IQ = 105 - 6.28 v1 + 0.68 u1v1

Predictor	Coef	StDev	T	P
Constant	105.318	2.493	42.25	0.000
v1	-6.282	2.493	-2.52	0.016
u1v1	0.682	2.493	0.27	0.786

S = 15.32 R-Sq = 15.4% R-Sq(adj) = 10.6%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	1495.2	747.6	3.18	0.054
Residual Error	35	8217.0	234.8		
Total	37	9712.2			

From this output, we know that $SSE_{full} = 5941.2$, $SSE_{w/o\ biol} = 8217.0$, $SSE_{w/o\ adopt} = 7218.6$, and $SSE_{w/o\ interact} = 5943.1$. Then

$$SSE_{biol} = 8217.0 - 5941.2 = 2275.8,$$
$$SSE_{adopt} = 7218.6 - 5941.2 = 1277.4,$$
$$SSE_{interact} = 5943.1 - 5941.2 = 1.9.$$

These match the adjusted SS from part (a).

13.14

Analysis of Variance

Source	SS	d.f.	MS	F	Sig.
Alcohol	4.234	2	2.117	1.06	0.377
Base	5.026	1	5.026	2.51	0.139
Interaction	23.944	2	11.972	5.98	0.016
Error	24.018	12	2.002		
Total	57.222	17			

Both Alcohol and Base are only significant as interaction effects, at $\alpha = 0.05$. These results are similar to the results of Exercise 13.2.

Solutions to Section 13.2

13.15 (a) The effect estimates are

$$A = \frac{(9-5)+(15-7)}{2} = 6,$$
$$B = \frac{(7-5)+(15-9)}{2} = 4,$$
$$AB = \frac{(15-7)-(9-5)}{2} = 2.$$

(b) The degrees of freedom are $2^k(n-1) = 2^2(3-1) = 8$, so the MSE is $s^2 = SSE/8 = 96/8 = 12$.

(c) The F statistics for each effect are

$$F_A = \frac{(n2^{k-2})A^2}{s^2} = \frac{3 \times 6^2}{12} = 9,$$
$$F_B = \frac{(n2^{k-2})B^2}{s^2} = \frac{3 \times 4^2}{12} = 4,$$
$$F_A = \frac{(n2^{k-2})(AB)^2}{s^2} = \frac{3 \times 2^2}{12} = 1.$$

Compare each of these to $F_{1,8,0.10} = 3.46$, and conclude that the AB interaction is not significant, but the A and B main effects are significant.

13.16

Chapter 14 Solutions

Solutions to Section 14.1

14.1 (a) Using the table below,

x_i	37	26	31	35	32	32	27	31	34	36
$\text{Sign}(x_i - 30)$	+	−	+	+	+	+	−	+	+	+

$s_+ = 8$ and $s_- = 2$. Then the exact P-value is

$$P = P(S \le s_-) = P(S \le 2) = 0.055.$$

To find the large-sample P-value, first compute

$$z = \frac{s_+ - n/2 - 1/2}{\sqrt{n/4}} = \frac{8 - 5 - 1/2}{\sqrt{10/4}} = 1.581.$$

Then the approximate P-value is

$$P \approx 1 - \Phi(1.581) = 0.057.$$

The normal approximation is quite accurate, differing by only 0.002. Since both P-values are $> \alpha = 0.05$, do not reject H_0 (although it is on the threshold).

(b) Using the table below,

x_i	37	26	31	35	32	32	27	31	34	36
d_i	+7	−4	+1	+5	+2	+2	−3	+1	+4	+6
r_i	10	6.5	1.5	8	3.5	3.5	5	1.5	6.5	9

$$w_+ = 10 + 1.5 + \ldots + 9 = 43.5 \text{ and } w_- = 6.5 + 5 = 11.5.$$

Then the exact P-value is

$$P = P(W \ge w_+) = P(W \ge 43.5) = P(W \ge 44) = 0.053.$$

To find the large-sample P-value, first compute

$$z = \frac{w_+ - n(n+1)/4 - 1/2}{\sqrt{n(n+1)(2n+1)/24}} = \frac{43.5 - 27.5 - 0.5}{\sqrt{10 \times 11 \times 21/24}} = 1.58.$$

Then the approximate P-value is

$$P \approx 1 - \Phi(1.58) = 0.057.$$

The normal approximation is slightly less accurate than with the Sign Test, differing by about 0.004. The conclusion is the same, namely, do not reject H_0 although it is on the threshold.

14.2 (a) Using the table below,

d_i	6	8	9	5	−7	5	3	3	−12	3	0	1
$\text{Sign}(d_i)$	+	+	+	+	−	+	+	+	−	+	0	+

$s_+ = 9$ and $s_- = 2$. Then the exact P-value is

$$P = P(S \le s_-) = P(S \le 2) = 0.0327$$

Since $P < \alpha = 0.10$, reject H_0 and conclude that vitamin B does improve the IQ.

(b) Using the table below,

d_i	6	8	9	5	−7	5	3	3	−12	3	0	1
r_i	7	9	10	5.5	8	5.5	3	3	11	3		1

$$w_+ = 7 + 9 + \ldots + 1 = 47 \text{ and } w_- = 8 + 11 = 19.$$

Then the exact P-value is

$$P = P(W \le w_-) = P(W \le 19) = 0.1201.$$

The Wilcoxon Signed Rank test is less significant because the ranks associated with the negative differences are large in absolute value, increasing w_-, decreasing w_+, and increasing the P-value of the test.

14.3 For $n = 11$ and $p = 0.5$, $b = 2$ is the lower 0.033 critical point of the binomial distribution. Then a 93.4% sign CI for $\tilde{\mu}$, using the treatment differences from Exercise 14.2, is given by

$$\left[x_{(b+1)}, x_{(n-b)}\right] = \left[x_{(3)}, x_{(9)}\right] = [1, 6].$$

For $n = 11$, $w = 13$ is the lower 0.0415 critical point for the distribution of the Wilcoxon signed rank statistic. The table of Walsh averages is below:

	−12	−7	1	3	3	3	5	5	6	8	9
−12	−12	−9.5	−5.5	−4.5	−4.5	−4.5	−3.5	−3.5	−3	−2	−1.5
−7		−7	−3	−2	−2	−2	−1	−1	−0.5	0.5	1
1			1	2	2	2	3	3	3.5	4.5	5
3				3	3	3	4	4	4.5	5.5	6
3					3	3	4	4	4.5	5.5	6
3						3	4	4	4.5	5.5	6
5							5	5	5.5	6.5	7
5								5	5.5	6.5	7
6									6	7	7.5
8										8	8.5
9											9

Then a 91.7% Wilcoxon signed rank CI for $\tilde{\mu}$ is

$$\left[\bar{x}_{(w+1)}, \bar{x}_{(N-w)}\right] = \left[\bar{x}_{(14)}, \bar{x}_{(53)}\right] = [-2, 5.5].$$

These confidence intervals agree with the results of the hypothesis test. The sign CI does not contain 0, so the median difference is significantly greater than 0. The Wilcoxon signed rank CI does contain 0, so the median difference is not significantly greater than 0.

14.4

(a) Using the table below,

d_i	-4	0	12	18	-4	-12	8	16
Sign(d_i)	$-$		$+$	$+$	$-$	$-$	$+$	$+$

$s_+ = 4$ and $s_- = 3$. Then the exact P-value is

$$P = 2P(S \le s_{\min}) = 2P(S \le 3) = 2 \times 0.5 = 1.0$$

Since $P > \alpha = 0.05$, do not reject H_0 and conclude that glaucoma does not affect corneal thickness.

(b) Using the table below,

d_i	-4	0	12	18	-4	-12	8	16
r_i	1.5		4.5	7	1.5	4.5	3	6

$$w_+ = 4.5 + 7 + 3 + 6 = 20.5 \text{ and } w_- = 1.5 + 1.5 + 4.5 = 7.5.$$

Then the exact P-value is

$$P = 2P(W \le w_{\min}) = P(W \le 7.5) = 0.1484.$$

Since $P > \alpha = 0.05$, the conclusion is the same as in (a), namely that glaucoma does not affect corneal thickness.

14.5 For $n = 7$ and $p = 0.5$, $b = 0$ is the lower 0.0078 critical point of the binomial distribution. Then a 98.44% sign CI for $\tilde{\mu}$, using the treatment differences from Exercise 14.4, is given by

$$\left[x_{(b+1)}, x_{(n-b)}\right] = \left[x_{(1)}, x_{(7)}\right] = [-12, 18].$$

For $n = 7$, $w = 2$ is the lower 0.0234 critical point for the distribution of the Wilcoxon signed rank statistic. The table of Walsh averages is below:

	-12	-4	-4	8	12	16	18
-12	-12	-8	-8	-2	0	2	3
-4		-4	-4	2	4	6	7
-4			-4	2	4	6	7
8				8	10	12	13
12					12	14	15
16						16	17
18							18

Then a 95.32% Wilcoxon signed rank CI for $\tilde{\mu}$ is

$$\left[\bar{x}_{(w+1)}, \bar{x}_{(N-w)}\right] = \left[\bar{x}_{(3)}, \bar{x}_{(26)}\right] = [-8, 16].$$

These confidence intervals agree with the results of the hypothesis test. Both confidence intervals contain 0, indicating a nonsignificant result, and that glaucoma does not affect corneal thickness.

14.6

$$u_1 = w_1 - \frac{n_1(n_1+1)}{2} = 72 - \frac{9(10)}{2} = 27,$$

$$u_2 = w_2 - \frac{n_2(n_2+1)}{2} = 64 - \frac{7(8)}{2} = 36.$$

Then, using $n_1 = 7$ and $n_2 = 9$, the P-value is

$$P = P(U \geq u_1) = P(U \leq u_2) = 0.3403.$$

Since the P-value is $> \alpha = 0.10$, do not reject H_0 and conclude that treatment does not prolong survival.

14.12 The table of the ranks is given below:

	Ranks									
Nonpsychotic	1	2	3	4	5	6	8	9	10	11
	12.5	12.5	16	19	21					
Psychotic	7	14	15	17	18	20	22	23	24	25

Then

$$w_1 = 1 + 2 + \ldots + 21 = 140,$$

$$w_2 = 7 + 14 + \ldots + 25 = 185,$$

$$u_1 = w_1 - \frac{n_1(n_1+1)}{2} = 140 - \frac{15(16)}{2} = 20,$$

$$u_2 = w_2 - \frac{n_2(n_2+1)}{2} = 185 - \frac{10(11)}{2} = 130.$$

The large sample approximation is

$$z = \frac{u_{\max} - n_1 n_2/2 - 1/2}{\sqrt{\frac{n_1 n_2 (N+1)}{12}}} = \frac{130 - (15)(10)/2 - 1/2}{\sqrt{\frac{(15)(10)(26)}{12}}} = 3.023.$$

The large sample P-value is

$$P = 2(1 - \Phi(3.023)) = 2 \times 0.0013 = 0.0026.$$

Since the P-value is $< \alpha = 0.05$, reject H_0 and conclude that psychotic and nonpsychotic patients have significantly different dopamine levels.

14.13 (a) The table of the ranks is given below:

	Ranks of Carbon Measurements									
Method 1	13	10	8	6	5	1	2	3	12	7
Method 2	15	14	9	4	11					

Then

$$w_1 = 13 + 10 + \ldots + 7 = 67,$$

$$w_2 = 15 + 14 + \ldots + 11 = 53,$$

$$u_1 = w_1 - \frac{n_1(n_1+1)}{2} = 67 - \frac{10(11)}{2} = 12,$$

$$u_2 = w_2 - \frac{n_2(n_2+1)}{2} = 53 - \frac{5(6)}{2} = 38.$$

Then, using $n_1 = 10$ and $n_2 = 5$, the P-value is

$$P = 2P(U \le u_{\min}) = 2P(U \le 12) = 2 \times 0.0646 = 0.1292.$$

Since the P-value is $> \alpha = 0.10$, do not reject H_0 and conclude that the methods are not significantly different.

(b) The table of the differences d_{ij} between method II and method I is given below:

	Method II				
Method I	12.0006	12.0069	12.0075	12.0246	12.0318
11.9853	0.0153	0.0216	0.0222	0.0393	0.0465
11.9949	0.0057	0.0120	0.0126	0.0297	0.0369
11.9985	0.0021	0.0084	0.0090	0.0261	0.0333
12.0016	−0.0010	0.0053	0.0059	0.0230	0.0302
12.0054	−0.0048	0.0015	0.0021	0.0192	0.0264
12.0061	−0.0055	0.0008	0.0014	0.0185	0.0257
12.0064	−0.0058	0.0005	0.0011	0.0182	0.0254
12.0072	−0.0066	−0.0003	0.0003	0.0174	0.0246
12.0077	−0.0071	−0.0008	−0.0002	0.0169	0.0241
12.0129	−0.0123	−0.0060	−0.0054	0.0117	0.0189

For $n_1 = 10$ and $n_2 = 5$, $u = 11$ is the 0.0496 critical point of the Wilcoxon-Mann-Whitney distribution. Then a 90.08% CI for $\tilde{\mu}_{II} - \tilde{\mu}_I$ is given by

$$\left[d_{(u+1)}, d_{(N-u)}\right] = \left[d_{(12)}, d_{(39)}\right] = [-0.0002, 0.0241].$$

This CI contains 0 and therefore agrees with the hypothesis test in (a).

14.14 The possible outcomes are enumerated below:

Ranks								Ranks							
1	2	3	4	5	6	w_1	u_1	1	2	3	4	5	6	w_1	u_1
x	x	x	y	y	y	6	0	y	y	y	x	x	x	15	9
x	x	y	x	y	y	7	1	y	y	x	y	x	x	14	8
x	x	y	y	x	y	8	2	y	y	x	x	y	x	13	7
x	x	y	y	y	x	9	3	y	y	x	x	x	y	12	6
x	y	x	x	y	y	8	2	y	x	y	y	x	x	13	7
x	y	x	y	x	y	9	3	y	x	y	x	y	x	12	6
x	y	x	y	y	x	10	4	y	x	y	x	x	y	11	5
x	y	y	x	x	y	10	4	y	x	x	y	y	x	11	5
x	y	y	x	y	x	11	5	y	x	x	y	x	y	10	4
x	y	y	y	x	x	12	6	y	x	x	x	y	y	9	3

Then the distribution of w_1 and u_1 is given in the table below:

w_1	u_1	$P(W_1 = w_1) = P(U_1 = u_1)$
6	0	0.05
7	1	0.05
8	2	0.10
9	3	0.15
10	4	0.15
11	5	0.15
12	6	0.15
13	7	0.10
14	8	0.05
15	9	0.05

The mean is

$$E(U_1) = 0 \times 0.05 + 1 \times 0.05 + \ldots + 9 \times 0.05 = 4.5,$$

and the variance is

$$\text{Var}(U_1) = \left[0^2 \times 0.05 + 1^2 \times 0.05 + \ldots + 9^2 \times 0.05\right] - (4.5)^2 = 5.25.$$

This agrees with the formulas for the mean and the variance of U_1.

14.15 (a) Out of the

$$\binom{n_1 + n_2}{n_1}$$

orderings (or rankings) of n_1 x's and n_2 y's, the ordering with all of the y's first, followed by all of the x's, results in the largest value of u_1, and hence the smallest P-value. This minimum P-value is given by

$$P_{\min} = P(U \geq u_1) = P(U = u_{\max}) = \frac{1}{\binom{n_1+n_2}{n_1}}.$$

(b) It is not possible to reject H_0 since the smallest P-value is

$$P = \frac{1}{\binom{n_1+n_2}{n_1}} = \frac{1}{\binom{8}{4}} = 0.014.$$

(c) For $n_1 = n_2 = 5$, the smallest P-value is

$$P = \frac{1}{\binom{n_1+n_2}{n_1}} = \frac{1}{\binom{10}{5}} = 0.004.$$

So for $n_1 = n_2 = 5$, rejection at $\alpha = 0.01$ is possible.

14.16 For $n_1 = 8$, $n_2 = 10$, and $\alpha = 0.051$, $u_{n_1,n_2,\alpha} = 59$. The approximation is

$$\begin{aligned} u_{n_1,n_2,\alpha} &\approx \frac{n_1 n_2}{2} + \frac{1}{2} + z_\alpha \sqrt{\frac{n_1 n_2 (N+1)}{12}} \\ &= \frac{(8)(10)}{2} + \frac{1}{2} + 1.635\sqrt{\frac{(8)(10)(18+1)}{12}} \\ &= 58.901 \text{ or } 59. \end{aligned}$$

The approximation is only off by 1.

14.17