(a) c satisfies

$$P(S > 5c|\sigma = 5) = 0.1$$

or

$$P\left(\chi^2_{19} = \frac{(n-1)S^2}{\sigma^2} > \frac{19 \times (5c)^2}{5^2}\right) = 0.1$$

or

$$P\left(\chi^2_{19} = 19c^2\right) = 0.1$$

or

$$19c^2 = \chi^2_{19,0.1} = 27.203,$$

so

$$c = \sqrt{\frac{27.203}{19}} = 1.197.$$

(b) Since $s = 7.5 > 5c = 5.983$, the engineer would conclude that $\sigma > 5$.

Solutions for Section 5.3

5.24 $t_{5,0.05} = 2.015$, $t_{10,0.10} = 1.372$, $t_{10,0.90} = -1.372$, and $t_{20,0.01} = 2.528$.

5.25 (a) $a = 1.812$, $b = -2.764$, $c = 1.372$, and $d = 2.228$.

(b) $a = t_{10,0.05}$, $b = t_{10,0.99} = -t_{10,0.01}$, $c = t_{10,0.10}$, and $d = t_{10,0.025}$.

5.26

$$P(-t_{8,.10} \le T_8 \le t_{8,.10}) = 0.80,$$

$$P(-t_{8,.05} \le T_8 \le t_{8,.01}) = 0.94,$$

$$P(t_{8,.05} \le T_8 \le t_{8,.01}) = 0.04,$$

$$P(T_8 > -t_{8,.05}) = 0.95.$$

5.27 (a) 100 random samples were generated.

(b) From the simulation, $t_{4,0.25} = -0.656$, $t_{4,0.5} = 0.081$, and $t_{4,0.90} = 2.065$. The exact values are $t_{4,0.25} = -0.741$, $t_{4,0.5} = 0$, and $t_{4,0.90} = 1.533$.

5.28 (a) 100 random samples were generated.

(b) From the simulation, $t_{4,0.25} = -0.554$, $t_{4,0.5} = 0.164$, and $t_{4,0.90} = 1.809$. The exact values are $t_{4,0.25} = -0.741$, $t_{4,0.5} = 0$, and $t_{4,0.90} = 1.533$.

Solutions for Section 5.4

5.29 $f_{10,10,0.025} = 3.72$, $f_{10,10,0.975} = 1/3.72 = 0.27$, $f_{5,10,0.10} = 2.52$, $f_{5,10,0.90} = 1/f_{10,5,0.10} = 0.30$, and $f_{10,5,0.90} = 1/2.52 = 0.40$.

5.30 (a) $a = 2.85$, $b = 0.176$, $c = 2.24$, $d = 0.24$, and $e = 3.51$.

(b) $a = f_{8,12,0.05}$, $b = f_{8,12,0.99} = 1/f_{12,8,0.01}$, $c = f_{8,12,0.10}$, $d = f_{8,12,0.975} = 1/f_{12,8,0.025}$, and $e = f_{8,12,0.025}$.

5.31

$$
\begin{aligned}
\alpha &= P\left(F_{\nu_1,\nu_2} \le f_{\nu_1,\nu_2,1-\alpha}\right) \\
&= P\left(\frac{1}{F_{\nu_2,\nu_1}} \le f_{\nu_1,\nu_2,1-\alpha}\right) \\
&= P\left(F_{\nu_2,\nu_1} \ge \frac{1}{f_{\nu_1,\nu_2,1-\alpha}}\right)
\end{aligned}
$$

Hence, $f_{\nu_2,\nu_1,\alpha} = 1/f_{\nu_1,\nu_2,1-\alpha}$.

5.32

$$
\begin{aligned}
1-\alpha &= P\left(-t_{\nu,\alpha/2} \le T_\nu \le t_{\nu,\alpha/2}\right) \\
&= P\left(T_\nu^2 \le t_{\nu,\alpha/2}^2\right) \\
&= P\left(F_{1,\nu} \le t_{\nu,\alpha/2}^2\right)
\end{aligned}
$$

Hence, $f_{1,\nu,\alpha} = t_{\nu,\alpha/2}^2$.

5.33

$$
\begin{aligned}
P\left(\frac{S_1^2}{S_2^2} > 4\right) &= P\left(\frac{\frac{(n_1-1)S_1^2}{\sigma^2}}{\frac{(n_2-1)S_2^2}{\sigma^2}} > 4\left[\frac{n_1-1}{n_2-1}\right]\right) \\
&= P\left(F_{n_1-1,n_2-1} > 4\left[\frac{n_1-1}{n_2-1}\right]\right)
\end{aligned}
$$

For $(n_1 = 7, n_2 = 5)$,

$$P\left(\frac{S_1^2}{S_2^2} > 4\right) = P\left(F_{6,4} > 6\right) \approx 0.05$$

For $(n_1 = 13, n_2 = 7)$,

$$P\left(\frac{S_1^2}{S_2^2} > 4\right) = P\left(F_{12,6} > 8\right) \approx 0.01$$

For $(n_1 = 9, n_2 = 16)$,

$$P\left(\frac{S_1^2}{S_2^2} > 4\right) = P\left(F_{8,15} > 2.133\right) \approx 0.10$$

Solutions for Section 5.5

5.34 Since the rth order statistic in a random sample of size n from a $U[0,1]$ distribution has t beta distribution with parameters r and $n-r+1$, $X_{\min} \sim \text{Beta}(1,9)$, $X_{\max} \sim \text{Beta}(9,1$ and $\tilde{X} = X_{(0.5)} \sim \text{Beta}(5,5)$.

5.35 From equation (5.26)

$$f_{(1)}(x) = \frac{1}{B(1,9)}\left(1-e^{-0.10x}\right)^0\left(e^{-0.10x}\right)^8\left(0.10e^{-0.10x}\right)$$

Chapter 6 Solutions

Solutions for Section 6.1

6.1 (a) 3 is a parameter, 0 is a statistic.

(b) 63 is a statistic.

(c) 48% is a statistic, 52% is a parameter.

6.2 (a)

$$\begin{aligned}
\text{Bias}(\hat{\mu_1}) &= E(\hat{\mu_1}) - \mu = E(X_1) - \mu = \mu - \mu = 0 \\
\text{Bias}(\hat{\mu_2}) &= E(\frac{X_2 + X_3}{2}) - \mu = \frac{E(X_2) + E(X_3)}{2} - \mu = \frac{2\mu}{2} - \mu = 0 \\
\text{Bias}(\hat{\mu_3}) &= E(0.1X_1 + 0.2X_2 + 0.3X_3 + 0.4X_4) - \mu = (0.1 + 0.2 + 0.3 + 0.4)\mu - \mu = 0 \\
\text{Bias}(\hat{\mu_4}) &= E(\bar{X}) - \mu = \mu - \mu = 0
\end{aligned}$$

(b)

$$\begin{aligned}
\text{Var}(\hat{\mu_1}) &= \sigma^2 \\
\text{Var}(\hat{\mu_2}) &= 0.5^2\sigma^2 + 0.5^2\sigma^2 = 0.5\sigma^2 \\
\text{Var}(\hat{\mu_3}) &= 0.1^2\sigma^2 + 0.2^2\sigma^2 + 0.3\sigma^2 + 0.4^2\sigma^2 = 0.3\sigma^2 \\
\text{Var}(\hat{\mu_4}) &= \text{Var}(\bar{X}) = \frac{\sigma^2}{4} = 0.25\sigma^2
\end{aligned}$$

Therefore $\hat{\mu_4} = \bar{X}$ has the smallest variance.

(c)

$$\text{Bias}(\hat{\mu}) = E(a_1X_1 + \cdots + a_nX_n) - \mu = (a_1 + \cdots + a_n)\mu - \mu = 0$$

So $\hat{\mu}$ is unbiased when $\sum a_i = 1$

$$\text{Var}(\hat{\mu}) = \text{Var}(a_1X_1 + \cdots + a_nX_n) = (a_1^2 + \cdots + a_n^2)\sigma^2 = (\sum a_i^2)\sigma^2$$

$\text{Var}(\hat{\mu})$ is minimum when $a_1 = \cdots = a_n = 1/n$ because subject to $\sum a_i = 1$, $\sum a_i^2$ is minimized by choosing $a_1 = \cdots = a_n = 1/n$.

6.3 (a)

$$\text{Bias}(X_{\max}) = E(X_{\max}) - \theta = \frac{n}{n+1}\theta - \theta = \frac{-1}{n+1}\theta.$$

(b)

$$\text{Bias}(X_{\max} + X_{\min}) = E(X_{\max} + X_{min}) - \theta = (\frac{1}{n+1} + \frac{n}{n+1})\theta - \theta = 0.$$

Since $X_{\max} + X_{\min}$ is an unbiased estimator of θ, $(X_{\max} + X_{\min})/2$ is an unbiased estimator of $\theta/2$.

6.4

$$\text{Bias}(\bar{X}^2) = E(\bar{X}^2) - \mu^2 = \text{Var}(\bar{X}) + \mu^2 - \mu^2 = \frac{\sigma^2}{n}$$

(a)

$$\begin{aligned} E(\hat{p}_1) &= E(\hat{p}) = p \\ E(\hat{p}_2) &= E(1/2) = 1/2 \\ \text{Bias}(\hat{p}_1) &= E(\hat{p}) - p = 0 \\ \text{Bias}(\hat{p}_2) &= E(1/2) - p = 1/2 - p \end{aligned}$$

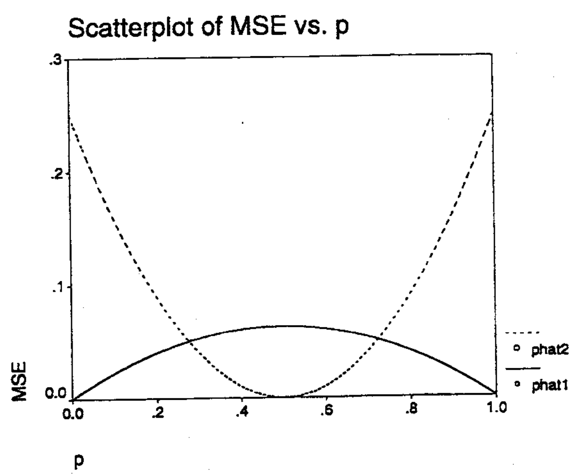
(b)

$$\begin{aligned} \text{Var}(\hat{p}_1) &= \text{Var}(\hat{p}) = \frac{p(1-p)}{n} \\ \text{Var}(\hat{p}_2) &= \text{Var}(1/2) = 0 \end{aligned}$$

$\hat{p}_2$ has the lower variance.

(c)

$$\begin{aligned} \text{MSE}(\hat{p}_1) &= \frac{p(1-p)}{n} + 0 = \frac{p(1-p)}{n} \\ \text{MSE}(\hat{p}_2) &= 0 + (1/2 - p)^2 = (1/2 - p)^2 \end{aligned}$$



$\hat{p}_1$ is generally a flatter curve, meaning there is less downside risk. However, $\hat{p}_2$ has a lower MSE for p near 0.5, so $\hat{p}_1$ is not always a better estimator.

6.6 (a)

$$\text{Bias}(\hat{\theta}) = w_1 E(\hat{\theta}_1) + w_2 E(\hat{\theta}_2) - \theta = (w_1 + w_2 - 1)\theta = 0.$$

Therefore $\hat{\theta}$ is unbiased if and only if $w_1 + w_2 = 1$.

(b)

$$\begin{aligned}\mathrm{Var}(\hat{\theta}) &= \mathrm{Var}(w_1\hat{\theta}_1 + w_2\hat{\theta}_2)\\ &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2\\ &= w^2\sigma_1^2 + (1-w)^2\sigma_2^2\end{aligned}$$

(c) Minimize $\mathrm{Var}(\hat{\theta}) = W = w^2\sigma_1^2 + (1-w)^2\sigma_2^2$:

$$\begin{aligned}\frac{\partial W}{\partial w} &= 2w\sigma_1^2 - 2(1-w)\sigma_2^2\\ &= w(2\sigma_1^2 + 2\sigma_2^2) - 2\sigma_2^2 = 0\end{aligned}$$

Therefore

$$w = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2} \text{ and } 1 - w = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}.$$

This says that $\hat{\theta}_1$ and $\hat{\theta}_2$ should be weighted inversely in proportion to their variances. Note :

$$\frac{\partial^2 W}{\partial w^2} = 2\sigma_1^2 + 2\sigma_2^2 > 0.$$

so that this value of w is a local minimum.

6.7

$$\bar{x} = (26.3 + 25.9 + \ldots + 26.2)/10 = 26.15$$

$$s^2 = [(26.3 - 26.15)^2 + (25.9 - 26.15)^2 + \ldots + (26.2 - 26.15)^2]/9 = 0.2317$$

Then

$$\mathrm{SEM} = \sqrt{\frac{0.2317}{10}} = 0.15.$$

6.8

$$SE(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(0.43)(0.57)}{611}} = 0.02.$$

6.9 For the gamma distribution we have

$$\mu = \frac{\theta_2}{\theta_1} \text{ and } \sigma^2 = \frac{\theta_2}{\theta_1^2}.$$

Equate

$$\hat{\mu} = \hat{\mu}_1 = \frac{\hat{\theta}_2}{\hat{\theta}_1} \text{ and } \hat{\sigma}^2 = \hat{\mu}_2 - \hat{\mu}_1^2 = \frac{\hat{\theta}_2}{\hat{\theta}_1^2},$$

where $\hat{\mu}_1$ and $\hat{\mu}_2$ are the first two sample moments. Solving these two equations yields

$$\hat{\theta}_1 = \frac{\hat{\mu}}{\hat{\sigma}^2} \text{ and } \hat{\theta}_2 = \frac{\hat{\mu}^2}{\hat{\sigma}^2}.$$

6.10 For the beta distribution we have

$$\mu = \frac{\theta_1}{\theta_1+\theta_2} \text{ and } \sigma^2 = \frac{\theta_1\theta_2}{(\theta_1+\theta_2)^2(\theta_1+\theta_2+1)}.$$

Equate

$$\hat{\mu} = \hat{\mu}_1 = \frac{\hat{\theta}_1}{\hat{\theta}_1+\hat{\theta}_2} \text{ and } \hat{\sigma}^2 = \hat{\mu}_2 - \hat{\mu}_1^2 = \frac{\hat{\theta}_1\hat{\theta}_2}{(\hat{\theta}_1+\hat{\theta}_2)^2(\hat{\theta}_1+\hat{\theta}_2+1)}.$$

where $\hat{\mu}_1$ and $\hat{\mu}_2$ are the first two sample moments. Solving the first equation yields

$$\hat{\theta}_1(1-\hat{\mu}) = \hat{\theta}_2\hat{\mu},$$

or

$$\hat{\theta}_2 = \hat{\theta}_1\frac{1-\hat{\mu}}{\hat{\mu}} = c\hat{\theta}_1,$$

where

$$c = \frac{1-\hat{\mu}}{\hat{\mu}}.$$

Solving the second equation yields

$$\hat{\sigma}^2 = \frac{c\hat{\theta}_1^2}{\hat{\theta}_1^2(1+c)^2\left[(1+c)\hat{\theta}_1+1\right]},$$

or

$$\hat{\theta}_1 = \frac{c-(1+c)^2\hat{\sigma}^2}{(1+c)^3\hat{\sigma}^2}.$$

Solutions for Section 6.2

6.11 (a) Since $P(-1.645 \le Z \le +1.645) = .90$, a 90% CI for μ is $[\bar{X} - 1.645\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.645\frac{\sigma}{\sqrt{n}}]$.

(b) 90% CI $= [30 - 1.645 \times \frac{10}{\sqrt{100}}, 30 + 1.645 \times \frac{10}{\sqrt{100}}] = [28.355, 31.645]$.

(c) The probability that μ is included in this CI is either 0 or 1 (not .90).

6.12 (a) From a simulation, all 25 samples contained the true mean 50, and 6 of the samples contained the wrong mean 53.

(b) The width of the 95% CI decreases when sample size increases from 20 to 100. You would expect the same number of intervals to contain the true mean 50, but fewer intervals to contain the wrong mean 53.

(c) If μ were unknown, you would not be able to tell if a CI contained μ.

6.13 (a) You would expect $100 \times 0.95 = 95$ intervals to contain the true mean 70.

(b) $X \sim \text{Bin}(100, 0.95)$.

6.14 (a) For 80% confidence, $z_{\alpha/2} = z_{.10} = 1.282$. Then the 80% CI is

$$16.3 \pm 1.282 \times \frac{6}{\sqrt{25}} = [14.76, 17.84].$$

For 90% confidence, $z_{\alpha/2} = z_{.05} = 1.645$. Then the 90% CI is

$$16.3 \pm 1.645 \times \frac{6}{\sqrt{25}} = [14.33, 18.27].$$

For 99% confidence, $z_{\alpha/2} = z_{.005} = 2.576$. Then the 99% CI is

$$16.3 \pm 2.576 \times \frac{6}{\sqrt{25}} = [13.21, 19.39].$$

(b) If n were increased to 100, the width of the CI would decrease by a factor of $\sqrt{100/25} = 2$.

6.15 (a) A 95% CI for $\mu = 110.5 \pm 1.96 \times \frac{3}{\sqrt{10}} = [108.64, 112.36]$.
Since this interval falls completely within the specification limits of [107.5,112.5], we can conclude that the specifications are met.

(b) Only the lower specific limit is critical, i.e., we want $\mu \geq 107.5$ volts. Calculate the lower one-sided 95% confidence bound :

$$\mu \geq \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} = 100.5 - 1.645 \times \frac{3}{\sqrt{10}} = 108.94.$$

Since this exceeds 107.5, the lower specification limit is met.

(c) Similarly if we want $\mu \leq 112.5$, calculate the upper one-sided 95% confidence bound :

$$\mu \leq \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} = 110.5 + 1.645 \times \frac{3}{\sqrt{10}} = 112.06.$$

Since this is smaller than 112.5, the upper specification limit is met.

6.16

$$\text{Conf. Level: } P(X_{\min} \leq \tilde{\mu} \leq X_{\max}) = 1 - \left(\frac{1}{2}\right)^{n-1}$$

For $n = 10$,

$$\text{Conf. Level: } P(X_{\min} \leq \tilde{\mu} \leq X_{\max}) = 1 - \left(\frac{1}{2}\right)^{10-1} = 0.998.$$

Solutions for Section 6.3