For 90% confidence, $z_{\alpha/2} = z_{.05} = 1.645$. Then the 90% CI is

$$16.3 \pm 1.645 \times \frac{6}{\sqrt{25}} = [14.33, 18.27].$$

For 99% confidence, $z_{\alpha/2} = z_{.005} = 2.576$. Then the 99% CI is

$$16.3 \pm 2.576 \times \frac{6}{\sqrt{25}} = [13.21, 19.39].$$

(b) If n were increased to 100, the width of the CI would decrease by a factor of $\sqrt{100/25} = 2$.

6.15 (a) A 95% CI for $\mu = 110.5 \pm 1.96 \times \frac{3}{\sqrt{10}} = [108.64, 112.36]$.
Since this interval falls completely within the specification limits of [107.5,112.5], we can conclude that the specifications are met.

(b) Only the lower specific limit is critical, i.e., we want $\mu \geq 107.5$ volts. Calculate the lower one-sided 95% confidence bound :

$$\mu \geq \bar{x} - z_\alpha \frac{\sigma}{\sqrt{n}} = 100.5 - 1.645 \times \frac{3}{\sqrt{10}} = 108.94.$$

Since this exceeds 107.5, the lower specification limit is met.

(c) Similarly if we want $\mu \leq 112.5$, calculate the upper one-sided 95% confidence bound :

$$\mu \leq \bar{x} + z_\alpha \frac{\sigma}{\sqrt{n}} = 110.5 + 1.645 \times \frac{3}{\sqrt{10}} = 112.06.$$

Since this is smaller than 112.5, the upper specification limit is met.

6.16

$$\text{Conf. Level: } P(X_{\min} \leq \tilde{\mu} \leq X_{\max}) = 1 - \left(\frac{1}{2}\right)^{n-1}$$

For $n = 10$,

$$\text{Conf. Level: } P(X_{\min} \leq \tilde{\mu} \leq X_{\max}) = 1 - \left(\frac{1}{2}\right)^{10-1} = 0.998.$$

Solutions for Section 6.3

6.17 (a) $H_0 : p = 0.54$ vs. $H_1 : p \neq 0.54$, where p is the actual proportion of voters who favor the congressman.

(b) $H_0 : p = 0.05$ vs. $H_1 : p < 0.05$, where p is the true proportion of overdue books.

(c) $H_0 : p = 0.40$ vs. $H_1 : p < 0.40$, where p is the true scrap rate.

(d) $H_0 : p = \frac{1}{2}$ vs. $H_1 : p \neq \frac{1}{2}$, where $p = P(\text{prefer Gatorade})$.

6.18 (a) $H_0 : \mu = 3.4$ vs. $H_1 : \mu > 3.4$, where μ is the mean fat content per yogurt cup.

(b) i. $H_0 : \mu = 10,000$ vs. $H_1 : \mu > 10,000$

ii. $H_0 : \mu = 10,000$ vs. $H_1 : \mu < 10,000$, where μ is the mean shear strength.

(c) $H_0 : \mu = 25$ vs. $H_1 : \mu < 25$, where μ is the mean commuting time.

(d) $H_0 : \mu = 0$ vs. $H_1 : \mu \neq 0$, μ is the mean difference in scores between Gatorade [illegible] Sport.

6.19 (a) Since it is more serious not to detect an unsafe food additive, the hypotheses [illegible] It is not safe in the amount normally consumed vs. H_1 : It is safe.

(b) Assuming it is more serious to not release an effective drug, the hypotheses [illegible] is not effective vs. H_1 : It is effective.

(c) Because of potential side effects, it is more important to reduce the chance of [illegible] an inequivalent drug on the market. Then the hypotheses are H_0 : It is not [illegible] vs. H_1 : It is equivalent.

(d) Because of possible unforeseen consequences of cloud seeding, it is more [illegible] reduce the chance of accepting cloud seeding as a technique when it isn't [illegible] the hypotheses are H_0 : It is not effective vs. H_1 : It is effective.

6.20 (a)

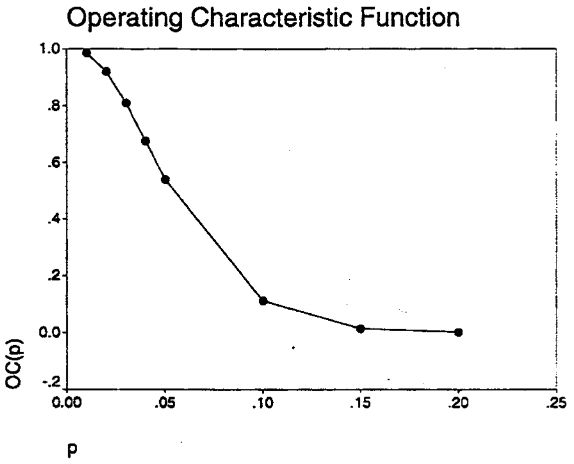
$$
\begin{aligned}
\alpha &= P(\text{Reject } H_0|H_0) = P(X = 1|p = 1/4) = 1/4 \\
\beta &= P(\text{Accept } H_0|H_1) = P(X = 0|p = 3/4) = 1/4
\end{aligned}
$$

(b)

$$
\begin{aligned}
\alpha &= P(X_1 + X_2 \neq 0 \text{ or } 1|H_0) = P(X_1 + X_2 = 2|H_0) = P(X_1 = 1 \text{ and } X_2 = [\text{illegible}]) \\
&= P(X_1 = 1|H_0)P(X_2 = 1|H_0) = (1/4) \times (1/4) = 1/16 \\
\beta &= P(X_1 + X_2 = 0 \text{ or } 1|H_1) = 1 - P(X_1 + X_2 = 2|H_1) \\
&= 1 - P(X_1 = 1 \text{ and } X_2 = 1|H_1) \\
&= 1 - (3/4) \times (3/4) \\
&= 7/16
\end{aligned}
$$

6.21

$$
OC(p) = P(X \leq 2) = \sum_{i=0}^{2} \binom{50}{i} p^i (1-p)^{50-i}
$$



$$\begin{aligned}
\alpha &= P(X > 2|p = 0.02) = 1 - \sum_{i=0}^{2} \binom{50}{i} (0.02)^i (0.98)^{50-i} = 1 - 0.922 = 0.078 \\
\beta &= P(X \le 2|p = 0.1) = \sum_{i=0}^{2} \binom{50}{i} (0.1)^i (0.9)^{50-i} = 0.112 \\
\text{Power}(0.15) &= P(X > 2|p = 0.15) = 1 - \sum_{i=0}^{2} \binom{50}{i} (0.15)^i (0.85)^{50-i} = 1 - 0.014 = 0.986.
\end{aligned}$$

6.22 (a) P-value $= P(X \ge 11|p = 1/2) = 1 - P(X \le 10|p = 1/2) = 0.059$. Since P-value $< \alpha = .10$, reject H_0.

(b) If H_1 is two-sided, P-value $= 2 \times 0.059 = 0.118$. Since $0.118 > \alpha = .10$, we cannot reject H_0.

6.23 (a) $\alpha = 0.05$. The test rejects H_0 when

$$\bar{X} > 0 + 1.645 \times \frac{1}{\sqrt{9}} = 0.548.$$

Then

$$\begin{aligned}
\beta &= P(\bar{X} \le c|\mu = 1) = P(\bar{X} \le 0.548|\mu = 1) \\
&= P\left(Z \le \frac{0.548 - 1}{1/\sqrt{9}}|\mu = 1\right) = P(Z \le -1.355) = 0.087
\end{aligned}$$

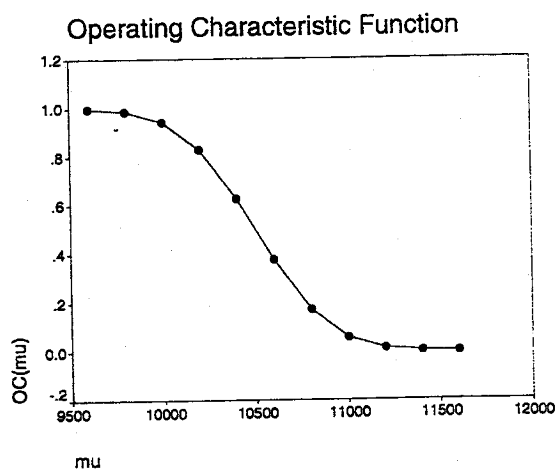
(b) In a simulation, a Type I error was committed 5% of the time. In a separate simulation, a Type II error was committed 5% of the time. α and β were both very close to the risks calculated in (a).

6.24

$$\begin{aligned}
\alpha &= P(\text{Reject } H_0|\mu = 10,000) = P(\bar{X} > 10,500|\mu = 10,000) \\
&= P(Z > \frac{10,500 - 10,000}{1000/\sqrt{10}}) = 1 - \Phi(1.581) = 0.0571 \\
\beta &= P(\text{Accept } H_0|\mu = 11,000) = P(\bar{X} \le 10,500|\mu = 11,000) \\
&= P(Z \le \frac{10,500 - 11,000}{1000/\sqrt{10}}) = \Phi(-1.581) = 0.0571
\end{aligned}$$

6.25

$$OC(\mu) = P(\bar{X} \le 10,500) = P\left(Z \le \frac{10,500 - \mu}{1000/\sqrt{10}}\right).$$



6.26 When $n = 10$:

$$\alpha = 0.01 = P(\bar{X} > c|\mu = 10,000) = P\left(Z > \frac{c - 10,000}{1000/\sqrt{10}}\right)$$

which is satisfied when

$$z_{0.99} = \frac{c - 10,000}{1000/\sqrt{10}} = 2.326$$

or $c = 10,735.55$.
Since the decision rule is to reject $H_0 : \mu = 10,000$ if $\bar{X} > 10,735.55$,

$$\beta = P\left(\bar{X} \le 10,735.55|\mu = 11,000\right) = \Phi\left(\frac{10,735.55 - 11,000}{1000/sqrt10}\right) = \Phi(-0.836) = \mathbf{0.2005}$$

When $n = 20$:

$$\alpha = 0.01 = P(\bar{X} > c|\mu = 10,000) = P\left(Z > \frac{c - 10,000}{1000/\sqrt{20}}\right)$$

(b) For the 2.5σ chart,

$$\begin{aligned}\beta &= P(15.888 \le \bar{X} \le 16.112|\mu = 16.1) \\ &= P\left(\frac{15.888 - 16.1}{0.1/\sqrt{5}} = -4.74 \le Z \le \frac{16.112 - 16.1}{0.1/\sqrt{5}} = 0.264\right) \\ &= 0.604.\end{aligned}$$

For the 3σ chart,

$$\begin{aligned}\beta &= P(15.866 \le \bar{X} \le 16.134|\mu = 16.1) \\ &= P\left(\frac{15.866 - 16.1}{0.1/\sqrt{5}} = -5.232 \le Z \le \frac{16.134 - 16.1}{0.1/\sqrt{5}} = 0.760\right) \\ &= 0.776.\end{aligned}$$

The 3 σ charts have a much higher β risk, while the α risk is not substantially lower.

6.30 (a)

$$\begin{aligned}P(\text{at least one true } H_0 \text{ rejected}) &= 1 - P(\text{No } H_0 \text{ rejected} \mid \text{all } H_0 \text{ are true}) \\ &= 1 - (0.95)^{20} = 1 - 0.358 \\ &= 0.642.\end{aligned}$$

(b) When testing multiple hypotheses, the probability of making at least one type I error is magnified above the desired level. An adjustment, either to the α used in the test or to the test procedure itself, is needed to compensate for this. Figuring out how to make these adjustments is the subject of the field of Multiple Comparisons.

6.31 For case (i),

$$z = \frac{0.1 - 0}{1/\sqrt{100}} = 1 \text{ and } P\text{-value} = 0.159.$$

For case (ii),

$$z = \frac{0.1 - 0}{1/\sqrt{400}} = 2 \text{ and } P\text{-value} = 0.023.$$

For case (iii),

$$z = \frac{0.1 - 0}{1/\sqrt{900}} = 3 \text{ and } P\text{-value} = 0.001.$$

As the sample size gets large, the P-value gets small, implying that even small differences from the hypothesized mean will be found to be significant if the sample size is large enough.

6.32 (a) From Exercise 5.44,

$$E(\hat{\theta}_1) = \frac{n}{n+1}\theta \text{ and } \mathrm{Bias}(\hat{\theta}_1) = \frac{1}{n+1}\theta.$$

Also, we know that the pdf of $\hat{\theta}_1$ is

$$f(\hat{\theta}_1) = n\left(\frac{\hat{\theta}_1}{\theta}\right)^{n-1}\frac{1}{\theta}.$$

7.1 (a) From equation (7.5),

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{2.326 \times 0.016}{0.005}\right]^2 = 55.41.$$

He needs to catch 56 fish.

(b) If $n = 100$, then

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 2.3263 \times \frac{0.016}{\sqrt{100}} = 0.0037.$$

The new margin of error is $\frac{0.0037}{0.005} = 0.74$ or 74% of the old margin of error.

7.2 (a) Range $= \pm 5$ psi $=$ 10psi. Therefore, a rough estimate of $\sigma = \frac{10}{4} = 2.5$ psi. Then from equation (7.5),

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.645 \times 2.5}{0.5}\right]^2 = 67.651.$$

He needs to test 68 fibers.

(b) From equation (7.2), a 90% CI for μ is given by:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 50 \pm 1.645 \times \frac{2.45}{\sqrt{25}} = [49.194, 50.806].$$

7.3 (a) From equation (7.5),

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \times 5}{1}\right]^2 = 96.04.$$

The personnel department needs to sample 97 employee files.

(b) From equation (7.2), a 90% CI for μ is given by:

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 6.3 \pm 1.96 \times \frac{4.57}{\sqrt{97}} = [5.391, 7.209].$$

7.4 (a) Range $= \$350 - \$10 = \$340$. Therefore, a rough estimate of $\sigma = \frac{340}{4} = \$85$. For a 95% CI with a margin of error of 10, using equation (7.5),

$$n = \left[\frac{z_{\alpha/2}\sigma}{E}\right]^2 = \left[\frac{1.96 \times 85}{10}\right]^2 = 277.55.$$

Therefore, 278 incorrect orders need to be sampled.

(b) For a 99% CI with a margin of error of 10,

$$n = \left[\frac{2.5758 \times 85}{10}\right]^2 = 479.36.$$

Therefore, 480 incorrect orders need to be sampled.

7.5

$$
\begin{aligned}
n &= \left[\frac{(z_{\alpha/2}+z_\beta)\sigma}{\delta}\right]^2 \\
&\approx \left[\frac{(1.96+1.28)\times 0.1}{0.1}\right]^2 \\
&\approx 10.498.
\end{aligned}
$$

Therefore, 11 cans should be sampled.

7.7 (a) The observed test statistic is

$$z = \frac{\bar{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{60758-60000}{1500/\sqrt{16}} = 2.021.$$

Since this is a 1-sided test, the P-value is

$$P = [1-\Phi(z)] = [1-\Phi(2.021)] = [1-0.9783] = 0.022.$$

Our conclusion is not to reject H_0 at level $\alpha = 0.01$ since the P-value $> \alpha$. There is not sufficient evidence that the mean tire life for this sample differs from the average tire life for the old tread design.

(b) If $\mu = 61000$, the power for this 1-sided test is, from equation (7.7),

$$
\begin{aligned}
\pi(61000) &= P(\text{reject } H_0 \mid \mu = 61000) \\
&= \Phi\left(-z_\alpha + \frac{(\mu-\mu_0)\sqrt{n}}{\sigma}\right) \\
&= \Phi\left(-2.326 + \frac{(61000-60000)\sqrt{16}}{1500}\right) \\
&= \Phi(0.3407) \\
&= 0.633.
\end{aligned}
$$

(c) To assure 90% power in detecting a mean wear of 61000 miles, use $\beta = 1-$ Power $= 0.1$. Then, from equation (7.10),

$$
\begin{aligned}
n &\approx \left[\frac{(z_\alpha+z_\beta)\sigma}{\delta}\right]^2 \\
&\approx \left[\frac{(2.326+1.282)\times 1500}{61000-60000}\right]^2 \\
&\approx 29.29.
\end{aligned}
$$

Therefore, 30 tires should be tested.

7.8 (a) The appropriate hypotheses are:

$$H_0 : \mu = 5 \text{ vs. } H_1 : \mu \neq 5$$

The test parameter μ represents the true mean pH of the compound being tested.

(b) The probability of not detecting a change of one σ is $\beta = 0.01$. Also, the difference to be detected is $\delta = 1 \times \sigma$. Then the formula for computing n is given by equation (7.11),

$$\begin{aligned} n &\approx \left[\frac{(z_{\alpha/2} + z_\beta)\sigma}{\delta}\right]^2 \\ &\approx \left[\frac{(1.645 + 2.326) \times \sigma}{1 \times \sigma}\right]^2 \\ &\approx 15.77. \end{aligned}$$

Therefore, 16 samples should be tested.

(c) The observed test statistic is

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{4.915 - 5}{0.2/\sqrt{16}} = 1.7.$$

Then the P-value is

$$P = 2[1 - \Phi(|z|)] = 2[1 - \Phi(1.7)] = 2[1 - 0.955] = 0.089.$$

Our conclusion is to reject H_0 at level $\alpha = 0.10$ since the P-value $< \alpha$. There is sufficient evidence that the mean pH has changed from the target value of 5.

7.9 (a) Since the consumer watchdog group suspects that the mean fat content *exceeds* 98% (or 3.4 grams per yogurt cup), this should be a 1-sided test. Therefore, the appropriate hypotheses are:

$$H_0 : \mu = 3.4 \text{ vs. } H_1 : \mu > 3.4,$$

where μ represents the mean fat content per yogurt cup.

(b) The observed test statistic is

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{3.6 - 3.4}{0.5/\sqrt{25}} = 2.0.$$

The P-value is

$$P = [1 - \Phi(z)] = [1 - \Phi(2.0)] = [1 - 0.977] = 0.023$$

Our conclusion is not to reject H_0 at level $\alpha = 0.01$ since the p-value $> \alpha$. There is not sufficient evidence to support the consumer group's claim that the mean fat content is higher than advertised.

(c) If $\mu = 3.7$, the power for this 1-sided test is, from equation (7.7),

$$\begin{aligned} \pi(3.7) &= P(\text{reject } H_0 \mid \mu = 3.7) \\ &= \Phi\left(-z_\alpha + \frac{(\mu - \mu_0)\sqrt{n}}{\sigma}\right) \\ &= \Phi\left(-2.326 + \frac{(3.7 - 3.4)\sqrt{25}}{0.5}\right) \\ &= \Phi(0.674) \\ &= 0.75. \end{aligned}$$

(a) You would expect 95 of the 95% z-intervals to contain the true mean $\mu = 12$.

(b) We still expect 95 of the 95% t-intervals to contain the true mean. The confidence intervals are developed so that 95% of them contain the true mean on average, regardless of the type of interval.

7.12 (a) A 90% z-interval is given by

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 68.45 \pm 1.645 \times \frac{3}{\sqrt{16}} = [67.216, 69.684].$$

(b) From equation (7.13), a 90% t-interval is given by

$$\bar{x} \pm t_{n-1,\alpha/2}\frac{\sigma}{\sqrt{n}} = 68.45 \pm 1.743 \times \frac{2.73}{\sqrt{16}} = [67.254, 69.646].$$

(c) The t-interval is shorter for this sample, but only because in this case s turned out to be less than σ. On the average, the z-intervals will be shorter since $z_{\alpha/2} < t_{n-1,\alpha/2}$ and $s \approx \sigma$. The z-intervals require more knowledge about the data and, because they utilize this extra information, are shorter than the t-intervals.

7.13 (a) Using $\bar{x} = 87.395$ and $s = 0.518$, a lower 95% confidence bound is given by

$$\bar{x} - t_{20-1,\alpha}\frac{s}{\sqrt{n}} = 87.395 \pm 1.729 \times \frac{0.518}{\sqrt{20}} = 87.195.$$

Since this lower confidence bound exceeds 87, we would conclude that the mean octane rating exceeds 87.

(b) The hypotheses are $H_0 : \mu \leq 87$ vs. $H_1 : \mu > 87$. The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{87.395 - 87}{0.518/\sqrt{20}} = 3.413.$$

Since $t_{19,0.005} = 2.861 < t < 3.579 = t_{19,0.001}$, the P-value lies between 0.005 and 0.001. Therefore this result would be significant at $\alpha = 0.005$ but not at $\alpha = 0.001$.

7.14 Taking into account the small sample size, we should perform a t-test. The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{4.915 - 5}{0.2/\sqrt{16}} = -1.7.$$

Since $|t| < t_{n-1,\alpha/2} = t_{15,.05} = 1.753$, our conclusion is not to reject H_0 at level $\alpha = 0.10$. There is not sufficient evidence that the mean pH level is different than the target value of 5.

The conclusion has changed because the t-test uses a larger critical value than the z-test, reducing the power. The t-test has less power because it assumes that σ is unknown, while the z-test assumes that σ is known.

7.15 (a) The parameter μ refers to the true average proportion of students using the food service.

(b) $\bar{x} = 68.5$ and $s = 6.66$. The test statistic then is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{68.5 - 60}{6.66/\sqrt{20}} = 5.77.$$

Since $|t| > t_{n-1,\alpha} = t_{19,.01} = 2.539$, our conclusion is to reject H_0 very strongly at level $\alpha = 0.01$. There is sufficient evidence that the mean usage of the food service has increased as of the 4th month of the contract.

(c) The appropriate hypotheses are

$$H_0 : \mu \leq 70 \text{ vs. } H_1 : \mu > 70.$$

(d) The test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{68.5 - 70}{6.66/\sqrt{20}} = -1.007.$$

Since $t < t_{n-1,\alpha} = t_{19,.10} = 1.328$, our conclusion is to not reject H_0 at level $\alpha = 0.10$. There is not sufficient evidence that the food service has met its goal of at least 70% usage.

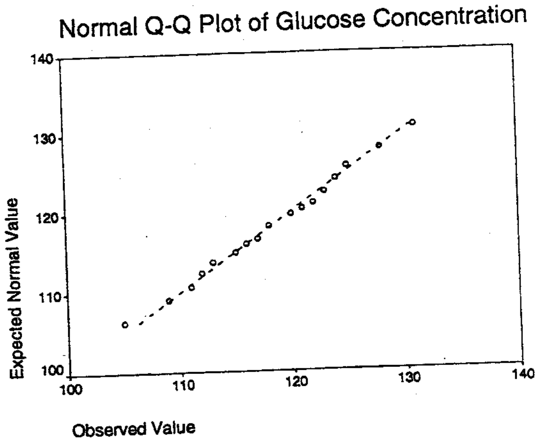
7.16 The hypotheses are $H_0 : \mu = 200$ vs. $H_1 : \mu \neq 200$. Using $\bar{x} = 201.770$ and $s = 2.410$, the test statistic is

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{201.770 - 200}{2.410/\sqrt{10}} = 2.322.$$

Since $|t| > t_{n-1,\alpha/2} = t_{9,.025} = 2.262$, our conclusion is to reject H_0 at level $\alpha = 0.05$. There is sufficient evidence that the mean thermostat setting is different than its design setting of 200 degrees.

Solutions for Section 7.3

7.17 (a)



This straight line normal plot indicates that the data follow a normal distribution.

(b) The test statistic is

$$\begin{aligned}\chi^2 &= \frac{(n-1)s^2}{\sigma_0^2} \\ &= \frac{(25-1)(6.2)^2}{(10)^2} \\ &= 9.226.\end{aligned}$$

Since $\chi^2 < \chi^2_{n-1,1-\alpha} = \chi^2_{24,0.90} = 15.659$, we reject the null hypothesis. There is sufficient evidence that the precision of the new device is better than the current monitor.

(c) An upper one-sided confidence interval for σ^2 is given by equation (7.20),

$$\begin{aligned}\sigma^2 &\le \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha}} \\ &\le \frac{24 \times (6.2)^2}{15.659} \\ &\le 58.916.\end{aligned}$$

Then an upper one-sided confidence bound for σ is $\sqrt{58.916} = 7.676$. Since this is less than $\sigma_0 = 10$, our conclusion is to reject H_0. This is consistent with our conclusion from the hypothesis test.

7.18 (a) The test statistic is

$$\begin{aligned}\chi^2 &= \frac{(n-1)s^2}{\sigma_0^2} \\ &= \frac{(16-1)(0.7)^2}{(1.0)^2} \\ &= 7.35.\end{aligned}$$

Since this χ^2 value falls between $\chi^2_{15,0.95} = 7.261$ and $\chi^2_{15,0.90} = 8.547$, the P-value falls between 0.05 and 0.10. The exact P-value is 0.053. There is not sufficient evidence that the precision of the new machine is better than the current machine.

(b) We want an upper one-sided confidence bound since the hypothesis test is to reject for low values of s^2. An upper one-sided confidence interval for σ^2 is given by equation (7.20)

$$\begin{aligned}\sigma^2 &\le \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha}} \\ &\le \frac{15 \times (0.7)^2}{7.261} \\ &\le 1.012.\end{aligned}$$

Then an upper one-sided confidence bound for σ is $\sqrt{1.012} = 1.006$. Since this is greater than $\sigma_0 = 1.0$, our conclusion is to not reject H_0. This is consistent with our conclusion from the hypothesis test.

7.19

(a) The nominal value for $\sigma = \frac{Range}{4} = \frac{82{,}000-68{,}000}{4} = 3500$. The appropriate hypotheses are:

$$H_0 : \sigma = 3500 \text{ vs. } H_1 : \sigma \neq 3500.$$

(b) A two-sided 95% confidence interval for σ is given by equation (7.18),

$$\begin{aligned} s\sqrt{\frac{n-1}{\chi^2_{n-1,\alpha/2}}} &\leq \sigma \leq s\sqrt{\frac{n-1}{\chi^2_{n-1,1-\alpha/2}}} \\ 4676\sqrt{\frac{24}{39.364}} &\leq \sigma \leq 4676\sqrt{\frac{24}{12.401}} \\ 3651 &\leq \sigma \leq 6505. \end{aligned}$$

Since the nominal value for σ of 3500 is not contained in this interval, we reject H_0. There is sufficient evidence to indicate that $\sigma > 3500$. To form a two-sided 99% confidence interval, use the critical values $\chi^2_{24,0.005}$ and $\chi^2_{24,0.995}$ instead of $\chi^2_{24,0.025}$ and $\chi^2_{24,0.975}$. Then the 99% confidence interval for σ is

$$[\sqrt{11518491}, \sqrt{53081067}] = [3394, 7286],$$

which includes $\sigma = 3500$. To summarize, at $\alpha = 0.05$ we can conclude that $\sigma > 3500$, but not at $\alpha = 0.01$.

Solutions for Section 7.4

7.20 Using $n = 200$, $\overline{x} = 1250$, and $s = 120$,

(a) A 95% confidence interval for the mean SAT score of all future students is given by

$$\begin{aligned} \overline{x} - t_{n-1,\alpha/2}s/\sqrt{n} &\leq \mu \leq \overline{x} + t_{n-1,\alpha/2}s/\sqrt{n} \\ 1250 - 1.96 \times 120/\sqrt{200} &\leq \mu \leq 1250 + 1.96 \times 120/\sqrt{200} \\ 1233.4 &\leq \mu \leq 1266.6. \end{aligned}$$

(b) A 95% prediction interval is wider since we are predicting a single future observation, X, and not the mean of all future observations. The formula is given by equation (7.22):

$$\begin{aligned} \overline{x} - t_{n-1,\alpha/2}s\sqrt{1+\frac{1}{n}} &\leq X \leq \overline{x} + t_{n-1,\alpha/2}s\sqrt{1+\frac{1}{n}} \\ 1250 - 1.96 \times 120 \times \sqrt{1+\frac{1}{200}} &\leq X \leq 1250 + 1.96 \times 120 \times \sqrt{1+\frac{1}{200}} \\ 1014.2 &\leq X \leq 1485.8. \end{aligned}$$

(c) A 95% tolerance interval is even wider than the prediction interval since we are searching for an interval that will contain a specified fraction (90%) of all future observations. The form of the tolerance interval, from equation (7.23), is $[\overline{x} - Ks, \overline{x} + Ks]$. Using $n = 200$, $1 - \alpha = 0.95$, and $\gamma = 0.90$, $K \approx 1.969$ from Table A.12 in the Appendix. Then the tolerance interval is

$$[1250 - 1.969 \times 120, 1250 + 1.969 \times 120] = [1013.72, 1486.28]$$

7.21 Using $n = 16$, $\overline{x} = 476.4$, and $s = 0.7$,

(a) A 95% confidence interval for the mean amount of beverage in a bottle is given by

$$\begin{aligned}
\bar{x} - t_{n-1,\alpha/2}s/\sqrt{n} &\leq \mu \leq \bar{x} + t_{n-1,\alpha/2}s/\sqrt{n} \\
476.4 - 2.131 \times 0.7/\sqrt{16} &\leq \mu \leq 476.4 + 2.131 \times 0.7/\sqrt{16} \\
476.03 &\leq \mu \leq 476.77.
\end{aligned}$$

(b) A 95% prediction interval is wider since we are predicting a single future observation, X, and not the mean of all future observations. The formula is given by equation (7.22):

$$\begin{aligned}
\bar{x} - t_{n-1,\alpha/2}s\sqrt{1+\frac{1}{n}} &\leq X \leq \bar{x} + t_{n-1,\alpha/2}s\sqrt{1+\frac{1}{n}} \\
476.4 - 2.131 \times 0.7 \times \sqrt{1+\frac{1}{16}} &\leq X \leq 476.4 + 2.131 \times 0.7 \times \sqrt{1+\frac{1}{16}} \\
474.86 &\leq X \leq 477.94.
\end{aligned}$$

(c) The 95% tolerance interval is even wider than the prediction interval since we are searching for an interval that will contain a specified fraction (95%) of all future observations. The form of the tolerance interval is $[\bar{x} - Ks, \bar{x} + Ks]$. Using $n = 16$, $1 - \alpha = 0.95$, and $\gamma = 0.95$, $K = 2.903$ from Table A.7 in the Appendix. Then the tolerance interval is

$$[476.4 - 2.903 \times 0.7, 476.4 + 2.903 \times 0.7] = [474.368, 478.432].$$

The tolerance interval does not fall within the specification limits: [475, 477]. This means that with 95% confidence, less than 95% of the bottles will fall within the specification limits. Therefore, the variability of the new filling machine is unacceptable.

7.22 Using $n = 25$, $\bar{x} = 74,283$, and $s = 4676$,

(a) A 95% prediction interval for the durability of this fabric is given by equation (7.22):

$$\begin{aligned}
\bar{x} - t_{n-1,\alpha/2}s\sqrt{1+\frac{1}{n}} &\leq X \leq \bar{x} + t_{n-1,\alpha/2}s\sqrt{1+\frac{1}{n}} \\
74283 - 2.064 \times 4676 \times \sqrt{1+\frac{1}{25}} &\leq X \leq 74283 + 2.064 \times 4676 \times \sqrt{1+\frac{1}{25}} \\
64441 &\leq X \leq 84125.
\end{aligned}$$

If an office needed a durability of at least 70,000 DR, then this fabric would not be a good purchase, since the lower prediction limit of 64441 DR is lower than the required durability.

(b) The form of the 95% tolerance interval for 99% of all fabric made by the same manufacturing process is, from equation (7.23), $\bar{x} - Ks, \bar{x} + Ks]$. Using $n = 25$, $1 - \alpha = 0.95$, and $\gamma = 0.99$, $K = 3.457$ from Table A.7 in the Appendix. Then the tolerance interval is

$$[74283 - 3.457 \times 4676, 74283 + 3.457 \times 4676] = [58118, 90448].$$

The tolerance interval does not fall within the specification limits; therefore the manufacturing process is unacceptable.

Solutions to Advanced Exercises

7.23