Chapter 8 Solutions

Solutions to Section 8.1

8.1 (a) Matched pairs.
(b) Independent samples.
(c) Matched pairs.
(d) Independent samples.

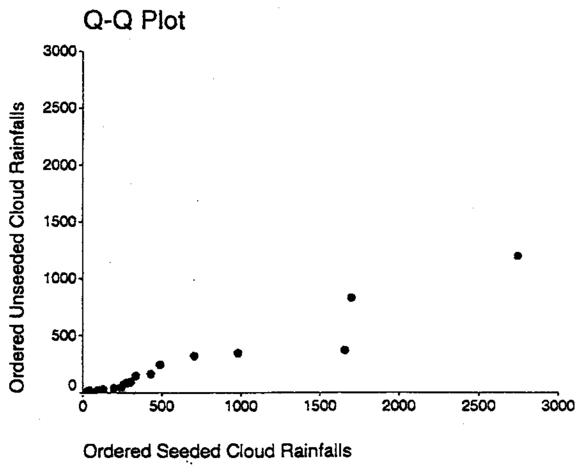
8.2 (a) Experimental.
(b) Observational.
(c) Experimental.
(d) Experimental.

8.3 (a) Matched pairs.
(b) Matched pairs.
(c) Independent samples.
(d) Independent samples.

8.4 (a) Observational.
(b) Observational.
(c) Observational.
(d) Experimental.

Solutions to Section 8.2

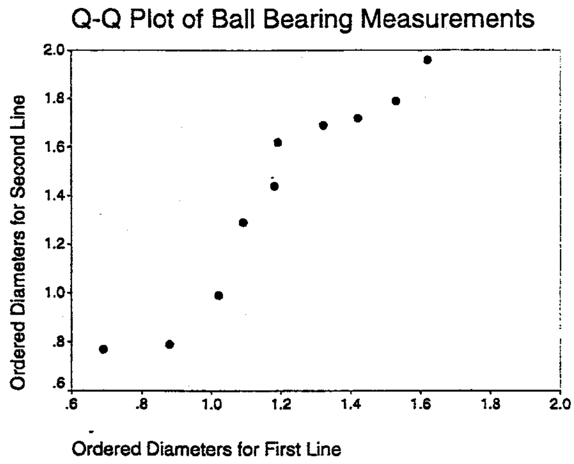
8.5 (a) The clouds in each group are not matched to one another on some characteristic.
(b)



Seeded cloud rainfalls tend to be larger than unseeded cloud rainfalls.

8.6 (a) The ball bearings from each line are not matched on any characteristic.

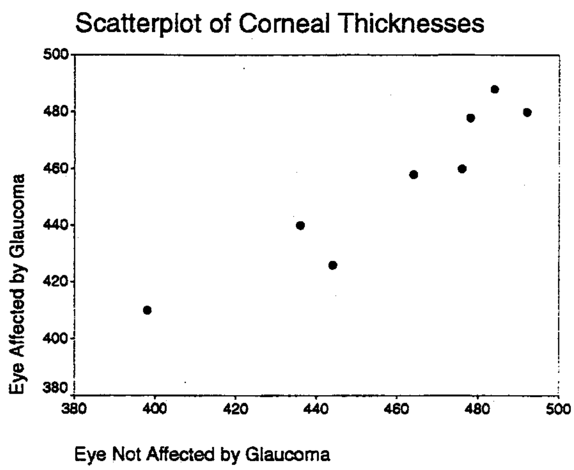
(b)



The second production line tends to have larger diameter ball bearings.

8.7 (a) The two different types of eyes are or belong to the same person.

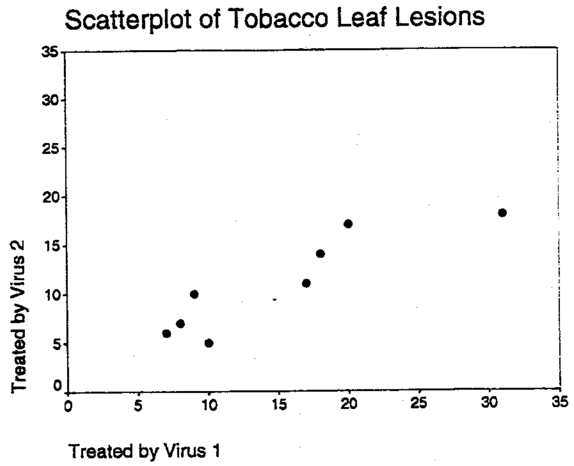
(b)



The pairs tend to lie pretty close to the 45 degree line through the origin. Eyes with glaucoma do not appear to have thicker corneas than unaffected eyes.

8.8 (a) The two viruses are applied to the same leaf, which makes each leaf a block.

(b)



The pairs tend to lie below the 45 degree line through the origin. Virus 1 tends to produce more lesions than virus 2.

Solutions to Section 8.3

8.9 (a) The pooled SD is

$$s = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(21-1)4.5+(21-1)2.0}{21+21-2}} = 1.803.$$

Then a 95% CI for $\mu_1 - \mu_2$ is given by:

$$\bar{x} - \bar{y} \pm t_{n_1+n_2-2,\alpha/2}\, s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 8 - 6.5 \pm 2.021 \times 1.803\sqrt{\frac{2}{21}} = [0.376, 2.625].$$

Since the CI does not contain 0, we reject H_0 and conclude that there is a significant difference between the two filters.

(b) Since

$$w_1 = \frac{s_1^2}{n_1} = \frac{4.5}{21} = 0.463 \text{ and } w_2 = \frac{s_2^2}{n_2} = \frac{2.0}{21} = 0.309,$$

the degrees of freedom are

$$\begin{aligned} \nu &= \frac{(w_1+w_2)^2}{w_1^2/(n_1-1)+w_2^2/(n_2-1)} \\ &= \frac{(0.463+0.309)^2}{(0.463)^2/(21-1)+(0.309)^2/(21-1)} \\ &= 34.845 \approx 35. \end{aligned}$$

Then a 95% CI for $\mu_1 - \mu_2$ is given by:

$$\bar{x} - \bar{y} \pm t_{\nu,\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 8 - 6.5 \pm 2.032 \times \sqrt{\frac{4.5}{21}+\frac{2.0}{21}} = [0.392, 2.608].$$

The results are similar, indicating that the assumption of equal variances is reasonable.

8.10

Using $\bar{x} = 0.0164$, $\bar{y} = 0.0243$, $s_1 = 0.0047$, $s_2 = 0.0051$, $n_1 = 15$, and $n_2 = 10$, the pooled SD is

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(15-1)(0.0047)^2 + (10-1)(0.0051)^2}{15 + 10 - 2}} = 0.0049.$$

Then a 95% CI for $\mu_1 - \mu_2$ is given by:

$$\begin{aligned} \text{95\% CI:} \quad &= \bar{x} - \bar{y} \pm t_{n_1+n_2-2,\alpha/2}\, s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \\ &= 0.0164 - 0.0243 \pm 2.069 \times 0.0049\sqrt{\frac{1}{15} + \frac{1}{10}} \\ &= [-0.0120, -0.0037]. \end{aligned}$$

Since the CI does not contain 0, we reject H_0 and conclude that there is a significant difference between the average dopamine levels of the two groups of patients.

8.11 (a) Using $\bar{x} = 12.0026$, $\bar{y} = 12.0143$, $s_1 = 0.0079$, $s_2 = 0.0132$, $n_1 = 10$, and $n_2 = 5$, the pooled SD is

$$s = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{(10-1)(0.0079)^2 + (5-1)(0.0132)^2}{10 + 5 - 2}} = 0.0099.$$

Then the test statistic is

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{12.0026 - 12.0143}{0.0099\sqrt{\frac{1}{10} + \frac{1}{5}}} = -2.1616.$$

Since $|t| > t_{10+5-2,\alpha/2} = 2.16$, we reject H_0 and conclude that there are significant differences between the methods.

(b) Since

$$w_1 = \frac{s_1^2}{n_1} = \frac{(0.0079)^2}{10} = (0.0025)^2 \text{ and } w_2 = \frac{s_2^2}{n_2} = \frac{(0.0132)^2}{5} = (0.0059)^2,$$

the degrees of freedom are

$$\begin{aligned} \nu \quad &= \frac{(w_1 + w_2)^2}{w_1^2/(n_1 - 1) + w_2^2/(n_2 - 1)} \\ &= \frac{((0.0025)^2 + (0.0059)^2)^2}{(0.0025)^4/(10-1) + (0.0059)^4/(5-1)} \\ &= 5.481 \approx 5. \end{aligned}$$

The test statistic is

$$t = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{12.0026 - 12.0143}{\sqrt{\frac{(0.0079)^2}{10} + \frac{(0.0132)^2}{5}}} = -1.817.$$

Since $|t| < t_{5,.025} = 2.571$, we do not reject H_0 at $\alpha = 0.05$. This is the opposite conclusion as that obtained from assuming equal variances.

(c) Since

$$w_1 = \frac{s_1^2}{n_1} = \frac{2.681}{26} = 0.103 \text{ and } w_2 = \frac{s_2^2}{n_2} = \frac{2.551}{26} = 0.098,$$

the degrees of freedom are

$$\begin{aligned}\nu &= \frac{(w_1+w_2)^2}{w_1^2/(n_1-1)+w_2^2/(n_2-1)}\\ &= \frac{(0.103+0.098)^2}{(0.103)^2/(26-1)+(0.098)^2/(26-1)}\\ &= 49.969 \approx 50.\end{aligned}$$

The test statistic is

$$t = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}} = \frac{3.995-5.136}{\sqrt{\frac{2.681}{26}+\frac{2.551}{26}}} = -2.544.$$

Since $|t| > t_{50,.025} \approx 2.011$, we still reject H_0. The conclusion is the same as that obtained from assuming equal variances, indicating that the assumption of equal variances is reasonable.

8.13 (a) Using $\bar{x} = 1.194$, $\bar{y} = 1.406$, $s_1^2 = 0.084$, $s_2^2 = 0.183$, and $n_1 = n_2 = 10$, the pooled SD is

$$s = \sqrt{\frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}} = \sqrt{\frac{(10-1)0.084+(10-1)0.183}{10+10-2}} = 0.366.$$

Then a 95% CI for $\mu_1 - \mu_2$ is given by:

$$\bar{x}-\bar{y} \pm t_{n_1+n_2-2,\alpha/2}\, s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} = 1.194-1.406 \pm 2.101 \times 0.366\sqrt{\frac{2}{10}} = [-0.556, 0.132].$$

Since the CI contains 0, we accept H_0 and conclude that there is no significant difference between the mean diameters of the two production lines.

(b) Since

$$w_1 = \frac{s_1^2}{n_1} = \frac{0.084}{10} = 0.0084 \text{ and } w_2 = \frac{s_2^2}{n_2} = \frac{0.183}{10} = 0.0183,$$

the degrees of freedom are

$$\begin{aligned}\nu &= \frac{(w_1+w_2)^2}{w_1^2/(n_1-1)+w_2^2/(n_2-1)}\\ &= \frac{(0.0084+0.0183)^2}{(0.0084)^2/(10-1)+(0.0183)^2/(10-1)}\\ &= 15.809 \approx 16.\end{aligned}$$

Then a 95% CI for $\mu_1 - \mu_2$ is given by:

$$\bar{x}-\bar{y} \pm t_{\nu,\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = 1.194-1.406 \pm 2.120 \times \sqrt{\frac{0.084}{10}+\frac{0.183}{10}} = [-0.560, 0.136].$$

The results are similar, indicating that the assumption of equal variances is reasonable.

8.14

(a) A 95% CI for $\mu_A - \mu_D$, where A =August and D =December, is given by

$$\bar{x}_A - \bar{x}_D \pm t_{n-1,\alpha/2} s_d/\sqrt{n} = 27.5 - 25.5 \pm 2.064 \times 2.0/\sqrt{24} = [1.157, 2.843].$$

Since this CI does not contain 0, we conclude that August had significantly higher IPM than December.

(b) A 95% CI for $\mu_A - \mu_D$ is given by

$$\bar{x}_A - \bar{x}_D \pm t_{n-1,\alpha/2} s_d/\sqrt{n} = 37.3 - 30.6 \pm 2.064 \times 13.0/\sqrt{24} = [1.223, 14.177].$$

Since this CI does not contain 0, we conclude that August had significantly higher PIT than December.

(c) Since both the number of items processed per minute and the percent of idle time are higher in August, one would conclude that the efficiency must be much higher during regular volume months.

8.15 (a) $H_0 : \mu_B - \mu_A = 0$ vs. $H_1 : \mu_B - \mu_A > 0$, where B =Before and A =After. Using $\bar{d} = 0.54$ and $s_d = 1.016$, the test statistic is

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{0.54 - 0}{1.016/\sqrt{10}} = 1.681.$$

Since $t > t_{10-1,0.10} = 1.383$, we reject H_0 at level $\alpha = 0.10$, and conclude that insulated houses have a lower energy consumption.

(b) Temperature: A warmer second winter could confound the insulation effect. Thermostat setting: If different settings are used, then the results could be affected.

8.16 (a) Using $\bar{d} = -4$ and $s_d = 10.744$, the test statistic is

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{-4 - 0}{10.744/\sqrt{8}} = -1.053.$$

Since $|t| < t_{8-1,0.05} = 1.895$, do not reject H_0 at level $\alpha = 0.10$, and conclude that the average corneal thicknesses are unaffected by glaucoma.

(b) A 90% CI for $\mu_1 - \mu_2$ is given by

$$\bar{d} \pm t_{n-1,\alpha/2} s_d/\sqrt{n} = -4 \pm 1.895 \times 10.744/\sqrt{8} = [-11.198, 3.198].$$

8.17 (a) Using $\bar{d} = 4$ and $s_d = 4.309$, the test statistic is

$$t = \frac{\bar{d} - 0}{s_d/\sqrt{n}} = \frac{4 - 0}{4.309/\sqrt{8}} = 2.625.$$

Since $t > t_{8-1,0.025} = 2.365$, we reject H_0 at level $\alpha = 0.05$, and conclude that the average number of lesions on tobacco leaves is different between the two types of viruses.

(b) A 95% CI for $\mu_1 - \mu_2$ is given by

$$\bar{d} \pm t_{n-1,\alpha/2} s_d/\sqrt{n} = 4 \pm 2.365 \times 4.309/\sqrt{8} = [0.397, 7.603].$$

Solutions to Section 8.4

8.18

The test statistic is

$$F = \frac{s_1^2}{s_2^2} = \frac{(2.3)^2}{(1.1)^2} = 4.372.$$

Since $F > f_{8,8,0.05} = 3.44$, reject H_0 and conclude that the new oven provides more even heating.

8.19 Using the log transformed data, $s_1^2 = 2.681$ and $s_2^2 = 2.551$. Also, $f_{25,25,0.025} = 2.230$ and $f_{25,25,0.975} = 1/2.230 = 0.448$. Then a 95% CI for σ_1^2/σ_2^2 is given by

$$\left[\frac{1}{f_{25,25,0.025}}\frac{s_1^2}{s_2^2}, \frac{1}{f_{25,25,0.975}}\frac{s_1^2}{s_2^2}\right] = \left[\frac{1}{2.230} \times \frac{2.681}{2.551}, \frac{1}{0.448} \times \frac{2.681}{2.551}\right] = [0.471, 2.344].$$

Since this interval contains 1, we can recommend using the pooled variance t-test.

8.20 The test statistic is

$$F = \frac{s_1^2}{s_2^2} = \frac{4.5}{2.0} = 2.25.$$

Since $F > f_{20,20,0.05} = 2.12$, reject H_0 and conclude that the variances are unequal. Therefore, one should use a t-test with separate variances to test the equality of means.

8.21 The sample variances are $s_1^2 = 0.000063$ and $s_2^2 = 0.000175$. Also, $f_{9,4,0.05} = 5.999$ and $f_{9,4,0.95} = 1/f_{4,9,0.05} = 0.275$. Then a 90% CI for σ_1^2/σ_2^2 is given by

$$\left[\frac{1}{f_{9,4,0.05}}\frac{s_1^2}{s_2^2}, \frac{1}{f_{9,4,0.95}}\frac{s_1^2}{s_2^2}\right] = \left[\frac{1}{5.999} \times \frac{0.000063}{0.000175}, \frac{1}{0.275} \times \frac{0.000063}{0.000175}\right] = [0.060, 1.308].$$

Since this interval contains 1, we can recommend using the pooled variance t-test. But the results using the pooled variances and separate variances t differed, so it would be better to use separate variances t.

8.22 The test statistic is

$$F = \frac{s_1^2}{s_2^2} = \frac{831}{592} = 1.404.$$

Since $F < f_{9,9,0.025} = 4.026$, do not reject H_0 and conclude that the variances are equal.

Solutions to Chapter 8 Advanced Exercises

8.23 (a) The pooled SD is, using H =High fiber diet and L =Low fiber diet,

$$s = \sqrt{\frac{(n_H - 1)s_H^2 + (n_L - 1)s_L^2}{n_H + n_L - 2}} = \sqrt{\frac{(20-1)(0.73)^2 + (20-1)(0.64)^2}{20 + 20 - 2}} = 0.686.$$

Then a 95% CI for $\mu_H - \mu_L$ is given by:

$$\bar{x}_H - \bar{x}_L \pm t_{n_H+n_L-2,\alpha/2}s\sqrt{\frac{1}{n_H} + \frac{1}{n_L}} = 4.44 - 4.46 \pm 2.021 \times 0.686\sqrt{\frac{2}{20}} = [-0.459, 0.419].$$

Treating the data as independent samples, the high and low fiber diets are not significantly different.

(b) Since the CI contains $p_0 = 0.10$, do not reject H_0 at $\alpha = 0.05$. The monitoring of the trainee should be continued at the same rate.

9.4 (a) With an a priori estimate of $p = 0.01$,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}^* \hat{q}^* = \left(\frac{1.96}{0.002}\right)^2 (0.01)(0.99) = 9507.96 \text{ or } 9508.$$

Therefore, 9508 parts should be sampled.
With no prior estimate of p,

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 \hat{p}^* \hat{q}^* = \left(\frac{1.96}{0.002}\right)^2 (0.5)(0.5) = 240100.$$

Therefore, 240,100 parts should be sampled.

(b) To ensure a sufficient number of defectives, sample the process until a specified number of defectives are found, so that n would be variable. This is called inverse sampling, as opposed to typical sampling where a fixed number of parts are selected.

9.5 (a) $H_0 : p = .465$ vs. $H_1 : p > .465$. The alternative is one-sided because the quarterback is interested only in improvement of his pass completion percentage.

(b) Using $\hat{p} = 82/151 = 0.543$, the test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0 / n}} = \frac{0.543 - 0.465}{\sqrt{(0.465)(0.535)/151}} = 1.923.$$

The P-value is

$$P = 1 - \Phi(1.923) = 0.0274.$$

Since $P < \alpha = 0.05$, reject H_0 and conclude that there is significant improvement in his pass completion percentage.

(c) Set

$$z_{0.025} = 1.96 = \frac{\hat{p} - 0.465}{\sqrt{(0.465)(0.535)/151}},$$

and solve for $\hat{p}$. This yields

$$\hat{p} = 0.465 + 1.96\sqrt{\frac{(0.465)(0.535)}{151}} = 0.5445.$$

He should have completed $n\hat{p} = 151 \times 0.5445 = 82.23$ or 83 completed passes to demonstrate siginificant improvement at $\alpha = 0.05$.

9.6 (a) For sensitivity, $\hat{p} = 0.8$ since 80 out of 100 high risk patients were correctly identified. Then a 90% CI for the sensitivity of the test is given by

$$\hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.8 \pm 1.645\sqrt{\frac{(0.8)(0.2)}{100}} = [0.734, 0.866].$$

(b) For specificity, $\hat{p} = 0.92$ since 184 out of 200 non high risk patients were correctly identified. Then a 90% CI for the specificity of the test is given by

$$\hat{p} \pm 1.645\sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.92 \pm 1.645\sqrt{\frac{(0.92)(0.08)}{200}} = [0.888, 0.952].$$

9.7 (a) $H_0 : p = .8$ vs. $H_1 : p > .8$.

(b) Using $\hat{p} = 46/50 = 0.92$, the test statistic is

$$z = \frac{\hat{p} - p_0}{\sqrt{p_0 q_0/n}} = \frac{0.92 - 0.8}{\sqrt{(0.8)(0.2)/50}} = 2.121.$$

The P-value is

$$P = 1 - \Phi(2{:}121) = 0.017.$$

Since $P < \alpha = 0.05$, reject H_0 and conclude that the new method is more accurate than the current method.

9.8 (a) If the true accuracy were $p = 0.9$, the power would be

$$\begin{aligned}\pi(0.9) &= \Phi\left[\frac{(p-p_0)\sqrt{n} - z_\alpha\sqrt{p_0 q_0}}{\sqrt{pq}}\right] \\ &= \Phi\left[\frac{(0.9-0.8)\sqrt{50} - 1.645\sqrt{(0.8)(0.2)}}{\sqrt{(0.9)(0.1)}}\right] \\ &= \Phi(0.164) = 0.5636.\end{aligned}$$

(b) To obtain a power of at least $1 - \beta = 0.75$,

$$\begin{aligned}n &= \left[\frac{z_\alpha\sqrt{p_0 q_0} + z_\beta\sqrt{p_1 q_1}}{\delta}\right]^2 \\ &= \left[\frac{1.645\sqrt{(0.8)(0.2)} + 0.675\sqrt{(0.9)(0.1)}}{0.9 - 0.8}\right]^2 \\ &= 74.05 \text{ or } 75.\end{aligned}$$

Therefore 75 patients should be tested.

9.9 For a power of at least $1 - \beta = 0.8$ in detecting a 2 percentage point shift from equally favored candidates,

$$\begin{aligned}n &= \left[\frac{z_{\alpha/2}\sqrt{p_0 q_0} + z_\beta\sqrt{p_1 q_1}}{\delta}\right]^2 \\ &= \left[\frac{1.96\sqrt{(0.5)(0.5)} + 0.84\sqrt{(0.48)(0.52)}}{0.02}\right]^2 \\ &= 4897.6 \text{ or } 4898.\end{aligned}$$

If 2500 voters are actually sampled, the power would be

$$\begin{aligned}\pi(0.52) &= \Phi\left[\frac{(p-p_0)\sqrt{n} - z_\alpha\sqrt{p_0 q_0}}{\sqrt{p_1 q_1}}\right] \\ &= \Phi\left[\frac{0.02\sqrt{2500} - 1.96\sqrt{(0.5)(0.5)}}{\sqrt{(0.52)(0.48)}}\right] \\ &= \Phi(0.040) = 0.5160.\end{aligned}$$

Solutions to Section 9.2

9.10 The hypotheses are

$$H_0 : p_1 = p_2 \text{ vs. } H_1 : p_1 \neq p_2.$$

A two-sided alternative is appropriate here because we are interested in detecting a change, but we do not have any prior knowledge about the direction of that change. Using $\hat{p}_1 = 0.40$, $\hat{p}_2 = 0.48$, $n_1 = n_2 = 1000$, the test statistic is

$$\begin{aligned} z &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}} \\ &= \frac{0.40 - 0.48}{\sqrt{\frac{(0.40)(0.60)}{1000} + \frac{(0.48)(0.52)}{1000}}} \\ &= -3.615. \end{aligned}$$

Since $|z| < z_{\alpha/2} = 1.96$, reject H_0 and conclude that there has been a change in opinion.

9.11 The hypotheses are

$$H_0 : p_1 = p_2 \text{ vs. } H_1 : p_1 \neq p_2.$$

A two-sided alternative is appropriate here because we are interested in detecting a change, but we do not have any prior knowledge of the direction of that change. Using $\hat{p}_1 = 17/482 = 0.0353$, $\hat{p}_2 = 29/503 = 0.0577$, $n_1 = 482$, and $n_2 = 503$, the test statistic is

$$\begin{aligned} z &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}} \\ &= \frac{0.0353 - 0.0577}{\sqrt{\frac{(0.0353)(0.9647)}{482} + \frac{(0.0577)(0.9423)}{503}}} \\ &= -1.674. \end{aligned}$$

The P-value is

$$P = 2(1 - \Phi(|-1.674|)) = 2 \times 0.0475 = 0.095.$$

Since $P < \alpha = 0.10$, reject H_0 and conclude that there has been a change in the proportion of students recognized as National Merit scholars.

9.12 The hypotheses are

$$H_0 : p_1 = p_2 \text{ vs. } H_1 : p1 \neq p_2.$$

For the vitamin C group, the proportion catching cold is $\hat{p}_1 = 17/139 = 0.122$. For the placebo group, the proportion catching cold is $\hat{p}_2 = 31/140 = 0.221$. Then the test statistic is

$$\begin{aligned} z &= \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}} \\ &= \frac{0.122 - 0.221}{\sqrt{\frac{(0.122)(0.878)}{139} + \frac{(0.221)(0.779)}{140}}} \\ &= -2.212. \end{aligned}$$

The P-value is

$$P = 2(1 - \Phi(|-2.212|)) = 2(0.0136) = 0.0272.$$

Since $P < \alpha = 0.05$, reject H_0 and conclude that taking vitamin C reduces the incidence rate of colds compared to a placebo.

9.13 (a) $H_0 : p_1 = p_2$ vs. $H_1 : p_1 \neq p_2$, where p_1 refers to the proportion of male faculty with an M.D. degree and p_2 refers to the proportion of female faculty with an M.D. degree. Use Fisher's exact test.

(b) Let X be the number of M.D.'s from the male sample and let Y be the number of M.D.'s from the female sample. The lower and upper P-values are given by

$$\begin{aligned}
P_L &= P(X \leq 5 | X + Y = 8) \\
&= \sum_{i=0}^{x} \frac{\binom{n_1}{i}\binom{n_2}{m-i}}{\binom{n}{m}} \\
&= \sum_{i=0}^{5} \frac{\binom{6}{i}\binom{9}{8-i}}{\binom{15}{8}} \\
&= \frac{9 + 216 + 1260 + 2520 + 1890 + 504}{6435} = 0.799. \\
P_U &= P(X \geq 5 | X + Y = 8) \\
&= \sum_{i=5}^{6} \frac{\binom{6}{i}\binom{9}{8-i}}{\binom{15}{8}} \\
&= \frac{504 + 36}{6435} = 0.084.
\end{aligned}$$

Then the P-value is

$$P = 2\min(p_L, p_U) = 2 \times 0.084 = 0.168.$$

Since $P > \alpha = 0.05$, do not reject H_0 and conclude that there is no significant difference between the proportions of male and female M.D.s.

9.14 (a) $H_0 : p_1 = p_2$ vs. $H_1 : p_1 \neq p_2$, where p_1 refers to the propotion of normal patients with low excretion and p_2 refers to the proportion of diabetic patients with low excretion. Use Fisher's exact test.

(b) Let X be the number with low excretion from the normal sample and let Y be the number with low excretion from the diabetic sample. The lower and upper P-values are given by

$$\begin{aligned}
P_L &= P(X \leq 10 | X + Y = 14) \\
&= \sum_{i=2}^{x} \frac{\binom{n_1}{i}\binom{n_2}{m-i}}{\binom{n}{m}} \\
&= \sum_{i=2}^{10} \frac{\binom{12}{i}\binom{12}{14-i}}{\binom{24}{14}} \\
&= \frac{66 + 2640 + \ldots + 3260}{1961256} = 0.999.
\end{aligned}$$

$$
\begin{aligned}
P_U &= P(X \geq 10 | X + Y = 14) \\
&= \sum_{i=10}^{12} \frac{\binom{12}{i}\binom{12}{14-i}}{\binom{24}{14}} \\
&= \frac{32670 + 2640 + 66}{1961256} = 0.018.
\end{aligned}
$$

Then the P-value is

$$P = 2\min(p_L, p_U) = 2 \times 0.018 = 0.036.$$

Since $P < \alpha = 0.05$, reject H_0 and conclude that there is a significant difference in the urinary thrombogobulin excretion between normal and diabetic patients.

9.15 (a) $H_0 : p_1 = p_2$ vs. $H_1 : p_1 \neq p_2$, where p_1 refers to the proportion of anesthetized patients using drug 1, and p_2 refers to the proportion of anesthetized patients using drug 2. Use McNemar's test.

(b) Using $m = b + c = 13 + 3 = 16$, the lower and upper one-sided P-values are

$$
\begin{aligned}
P_L &= P(C \leq 3 | B + C = 16) \\
&= \left(\frac{1}{2}\right)^{16} \sum_{i=0}^{b} \binom{m}{i} \\
&= \left(\frac{1}{2}\right)^{16} \sum_{i=0}^{3} \binom{16}{i} \\
&= \frac{1 + 16 + 120 + 560}{2^{16}} = 0.0106. \\
P_U &= P(C \geq 3 | B + C = 16) \\
&= \left(\frac{1}{2}\right)^{16} \sum_{i=3}^{16} \binom{16}{i} \\
&= 1 - \left(\frac{1}{2}\right)^{16} \sum_{i=0}^{2} \binom{16}{i} \\
&= 1 - \frac{1 + 16 + 120}{2^{16}} = 0.997.
\end{aligned}
$$

Then the two-sided P-value is

$$P = 2\min(P_L, P_U) = 2 \times 0.0106 = 0.021.$$

Since $P < \alpha = 0.05$, reject H_0 and conclude that there is a statistically significant difference between the two drugs.

9.16 (a) $H_0 : p_1 = p_2$ vs. $H_1 : p_1 \neq p_2$, where p_1 refers to the proportion of Pro students before the debate and p_2 refers to the proportion of Pro students after the debate. Use McNemar's test.

(b) Using $m = b + c = 8 + 26 = 34$, the large sample test statistic is

$$z = \frac{b-c-1}{\sqrt{b+c}} = \frac{8-26-1}{\sqrt{8+26}} = -3.258.$$

Then the P-value is

$$P = 2(1 - \Phi(|-3.258|)) = 2 \times 0.0006 = 0.0012.$$

Since $P < \alpha = 0.05$, reject H_0 and conclude that there was a change in opinion of the students.

Solutions to Section 9.3

9.17 $H_0 : p_1 = p_2 = \ldots = p_8 = 1/8$ vs. H_1 : Not H_0. The expected frequencies in each case are $np = 144 \times 1/8 = 18$. Then

$$\chi^2 = \sum \frac{(\text{Obs} - \text{Exp})^2}{\text{Exp}} = \frac{(29-18)^2}{18} + \frac{(19-18)^2}{18} + \ldots + \frac{(11-18)^2}{18} = 16.333.$$

Since $\chi^2 > \chi^2_{8-1,0.05} = 14.067$, reject H_0 and conclude that the horse's chances of winning are not the same for each starting gate.

9.18 $H_0 : p_1 = p_2 = \ldots = p_{12} = 1/12$ vs. H_1 : Not H_0. The expected frequencies in each case are $np = 700 \times 1/12 = 58.333$. Then

$$\chi^2 = \sum \frac{(\text{Obs} - \text{Exp})^2}{\text{Exp}} = \frac{(66-58.333)^2}{58.333} + \frac{(63-58.333)^2}{58.333} + \ldots + \frac{(42-58.333)^2}{58.333} = 19.726.$$

Since $\chi^2 > \chi^2_{12-1,0.05} = 19.675$, reject H_0 and conclude that the first births are not spread uniformly throughout the year.

9.19 (a) $H_0 : p_i = \binom{7}{i}(.5)^7$ vs. H_1 : Not H_0. Using

$$e_i = np_i = 98\binom{7}{i}(0.5)^7,$$

the results are summarized below:

Sons	n_i	e_i	$\frac{(n_i-e_i)^2}{e_i}$
0	0	0.766	
1	6	5.359	0.003
2	14	16.078	0.269
3	25	26.797	0.120
4	21	26.797	1.254
5	22	16.078	2.181
6	9	5.359	2.452
7	1	0.766	
Total	98	$\chi^2 =$	6.278