1. Recall $0 < m(E) < \infty$ and assume $0 < \alpha < 1$. If no such open interval $I$ exists, then $m(E \cap I) \leq \alpha m(I)$ for all open intervals $I = (a, b)$. Since $m(\{x\}) = 0$ for Lebesgue measure, the same holds for cells $I = (a, b]$. Choose $\epsilon > 0$ such that $\alpha(1 + \epsilon) < 1$ and then choose disjoint cells $I_j = [a_j, b_j]$ such that $E \subseteq B = \bigcup_{j=1}^{\infty} I_j$ and $\sum_{j=1}^{\infty} m(I_j) = m(B) < m(E) + m(E)\epsilon = m(E)(1+\epsilon)$. Then $m(E \cap I_j) \leq \alpha m(I_j)$ for all $j$ and $\sum_{j=1}^{\infty} m(E \cap I_j) = m(E \cap B) = m(E) \leq \alpha \sum_{j=1}^{\infty} m(I_j) = \alpha m(B) \leq \alpha(1 + \epsilon)m(E) < m(E)$, which contradicts $m(E) > 0$.

2. Choose disjoint cells $I_j = (a_j, b_j]$ such that $E \subseteq B = \bigcup_{j=1}^{\infty} I_j$ and $\mu(B) < \mu(E) + \epsilon/2$. Then $\mu(B - E) = \mu(B) - \mu(E) < \epsilon/2$. Let $B_n = \bigcup_{j=1}^{n} I_j$ where $\sum_{j=n+1}^{\infty} \mu(I_j) < \epsilon/2$. Then $\mu(B_n - E) \leq \mu(B - E) < \epsilon/2$ and $E - B_n \subseteq B - B_n = \bigcup_{j=n+1}^{\infty} I_j$ so that $\mu(E - B_n) < \epsilon/2$. Since $E \cap B_n = (E - B_n) \cup (B_n - E)$, we have $\mu(E \cap B_n) = \mu(E - B_n) + \mu(B_n - E) < \epsilon$.

3. Since $a_i \leq b_i$, the differences $c_i = b_i - a_i \geq 0$. By assumption, $\sum_{i=1}^{n} c_i = \sum_{i=1}^{n} (b_i - a_i) = \sum_{i=1}^{n} b_i - \sum_{i=1}^{n} a_i = 0$. If any $c_i > 0$, then $\sum_{j=1}^{n} c_j \geq c_i > 0$, which would be a contradiction, so that $c_i = 0$ (and $a_i = b_i$) for $1 \leq i \leq n$.

4. (i) $F(x)$ has jumps of size $1$ ($F(x+) - F(x-) = 1$) at all integers and is otherwise continuous. Thus $\mu_F(\{n\}) = 1$ and $\mu_F(\{a\}) = 0$ at all other values $a \in R$.

(ii) Since $F(n-) - F(n-1) = 0$ for all $n$, $\mu_F((n,n+1)) = 0$ for all open intervals $(n,n+1)$. By (i), $\mu_F(\{n\}) = 1$ for all integers $n$. The set $A$ contains intervals around the points $1/2, 1, 3/2, 2, 5/2$ and, in particular, contains the points $1, 2$ along with subsets of the open intervals $(0,1), (1,2), (2,3)$ (which are $\mu_F$-null sets). Thus $\mu_F(A) = \mu_F(\{1\}) + \mu_F(\{2\}) = 2$.

5. (i) Note (a) $\phi \in \Gamma_Q$, (b) $C_1, C_2 \in \Gamma_Q$ implies $C_1 \cap C_2 \in \Gamma_Q$, and (c) $C_1, C_2 \in \Gamma_Q$ implies $C_1 - C_2 = \bigcup_{j=1}^{m} D_j$ for disjoint $D_j \in \Gamma_Q$ and $m = 0, 1, 2$ by the same arguments as in the case of cells for dimension $k = 1$. The set function $\nu(A)$ is finitely additive on $\Gamma_Q$ for the same reason.

(ii) For $x \in Q$, $\nu^+(\{x\}) = \inf\{\sum_{i=1}^{\infty} \nu((a_i,b_i]_Q) : x \in \bigcup_{j=1}^{\infty} (a_i,b_i]_Q\}$. In particular $x \in (x-1/n, x+1/n]_Q$ for $n \geq 1$ implies

$$\nu^+(\{x\}) \leq \nu((x-1/n, x+1/n]_Q) = 2/n$$
for all \( n \geq 1 \), which implies \( \nu^*(\{x\}) = 0 \).

(iii) Every \( E \subseteq \mathbb{Q} \) is countable, so that, if \( E = \{q_n\} \) for \( q_n \in E \), then \( \nu^*(E) \leq \sum_{n=1}^{\infty} \nu^*(\{q_n\}) = 0 \).

(iv) If \( \nu \) were a premeasure on \( \Gamma_{\mathbb{Q}} \), then Proposition 1.13 would imply that \( \nu^*([a,b]_{\mathbb{Q}}) = \nu((a,b]_{\mathbb{Q}}) = b - a \), which is false by (iii) if \( a < b \). Thus \( \nu \) cannot be a premeasure. The proof of Proposition 1.15 that a set function like \( \nu \) is a premeasure requires that closed and bounded intervals be compact, which is not true in \( \mathbb{Q} \). Thus the proof of Proposition 1.15 breaks down at the step requiring compactness. Further steps that assume this are generally false, but all other steps in the proofs seem OK.