Math 310 Fall, 2020
Krantz

PRACTICE EXAM FOR FINAL EXAM

(6 points) **1.** Are the statements $(\sim \mathbf{A} \vee \sim \mathbf{B}) \vee (\mathbf{B} \wedge \mathbf{C})$ and $(\mathbf{A} \wedge \mathbf{B}) \Rightarrow (\mathbf{B} \wedge \mathbf{C})$ logically equivalent?

(6 points) **2.** Draw a Venn diagram to illustrate the identity

$$S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U)\,.$$

(9 points) **3.** Which of these functions is one-to-one? Which is onto? Give a brief reason for each answer.

(a) $f : \mathbb{R} \to \mathbb{R}$ $\qquad f(x) = x^2 - 2x$

(b) $g : \mathbb{Z} \to \mathbb{Z}$ $\qquad g(n) = n(n+2)$

(c) $h : \mathbb{R} \to \mathbb{R}$ $\qquad h(x) = x^2 \sin x$

(9 points) **4.** Which of these sets is countable and which uncountable (give a brief reason for each answer)?

(a) $\mathbb{C} \setminus \mathbb{Z}$

(b) $\mathbb{Z} \times \mathbb{C}$

(c) $\mathbb{Q} \times \mathbb{N}$

(6 points) **5.** Prove that is impossible for two perfect cubes to differ by 3.

(6 points) **6.** Use mathematical induction to prove that

$$1 + 2 + \cdots n = \frac{n(n+1)}{2}.$$

(8 points) **7.** Construct a Cantor-like set that has length 1/2.

(6 points) **8.** Prove that $\sqrt{8}$ is irrational.

(8 points) **9.** Use the least upper bound property of the reals to prove that 2 has a square root.

(8 points) **10.** What is the multiplicative inverse of the complex number $-i$?

(8 points) **11.** Find all cube roots of the complex number i.

(7 points) **12.** Let X be the integers $\mathbb{Z}$. Declare a set $U \subset X$ to be open if its complement is infinite. Does this define a topology?

(7 points) **13.** Construct a C^2 function from $\mathbb{R}$ to $\mathbb{R}$ which is positive for $x > 1$, positive for $x < -1$, and equal to 0 for $-1 \leq x \leq 1$.

(6 points) **14.** What is the multiplicative inverse of the quaternion $\mathbf{j} - \mathbf{k}$?