

MATH 310 SOLUTIONS TO HW 1

1 a)

| S | T | $S \wedge T$ | $S \vee T$ | $\sim(S \vee T)$ | $(S \wedge T) \vee \sim(S \vee T)$ |
|---|---|--------------|------------|------------------|------------------------------------|
| T | T | T | T | F | T |
| T | F | F | T | F | F |
| F | T | F | T | F | F |
| F | F | F | F | T | T |

c)

| S | T | $\sim S$ | $\sim T$ | $\sim S \vee T$ | $S \wedge \sim T$ | $\sim(S \wedge \sim T)$ | $(\sim S \vee T) \Leftrightarrow \sim(S \wedge \sim T)$ |
|---|---|----------|----------|-----------------|-------------------|-------------------------|---|
| T | T | F | F | T | F | T | T |
| T | F | F | T | F | T | F | T |
| F | T | T | F | T | F | T | T |
| F | F | T | T | T | F | T | T |

g)

| S | T | $\sim S$ | $T \vee \sim S$ | $S \wedge (T \vee \sim S)$ |
|---|---|----------|-----------------|----------------------------|
| T | T | F | T | T |
| T | F | F | T | T |
| F | T | T | T | F |
| F | F | T | T | F |

2 b) $V \Rightarrow (T \vee \sim S)$

d) $S \Rightarrow V$

f) $S \Rightarrow \sim T$

4, b) If there are clouds then it will rain

CONVERSE: If it will rain, then it is cloudy.

CONTRAPOSITIVE: If it is not raining, then there are no clouds.

d) CONVERSE: If men eat oats, then I am not a fool.

CONTRAPOSITIVE: If men do not eat oats, then I am a fool.

f) IF all people disarm then there will be peace in the world.

CONVERSE: If there is peace in the world, then all people will disarm.

CONTRAPOSITIVE: If there is no peace in the world, then some people will not disarm.

5 b) TRUE because both $2+2=4$ and $\frac{3}{5}$ is rational.

d) $2 \cdot 3 \neq 5$ so hypothesis is false.

The world is not flat so conclusion is false.

Hence implication is true.

f) $3^2 = 9$ so hypothesis is false.

$4^2 \neq 17$ so conclusion is false.

Hence implication is true.

6. b) $\sim (S \vee T)$

d) $\sim (\sim S \vee \sim (T \vee \sim S))$

10 b) There exists a first variable so that, for all second variables, if the second variable exceeds the first variable, then the second variable is greater than 5.

d) There is a variable which is positive and whose square exceeds its cube.

f) There is a variable so that it is not true that if the square of the variable is positive then the variable is positive.

14. b) $\sim A \Rightarrow \sim B$ is logically equivalent to $A \vee \sim B$. This is not logically equivalent to $A \wedge \sim B$.

d)

| A | B | $\sim A$ | $B \Rightarrow \sim A$ | $A \vee B$ | $A \Rightarrow (A \vee B)$ |
|---|---|----------|------------------------|------------|----------------------------|
| T | T | F | F | T | T |
| T | F | F | T | T | T |
| F | T | T | T | T | T |
| F | F | T | T | F | T |

not the same so logically inequivalent

f) $\sim(A \vee \sim B)$ is logically equivalent to $\sim A \wedge B$

22. $\forall x, P(x)$ says that $P(x)$ is true for every value of x .

$\sim \exists x, \sim P(x)$ says that it is not true that there exists an x for which $P(x)$ fails.

These say the same thing.