Math 320 Solutions to HW 3

pp. 79–83

1. b) $S = \{1, 2, 3, 4, 5\}$, $T = \{3, 4, 5, 7, 8, 9\}$,
$U = \{1, 2, 3, 4, 9\}$, $V = \{2, 4, 6, 8\}$

$(S \cap T) \cup U = \{3, 4, 5\} \cup \{1, 2, 3, 4, 9\}$
$= \{1, 2, 3, 4, 5, 9\}$

c) $(S \cup U) \cap V = \{1, 2, 3, 4, 5, 9\} \cap \{2, 4, 6, 8\}$
$= \{2, 4\}$.

f) $(S \cup V) \setminus (T \cap U) = \{1, 2, 3, 4, 5, 6, 8\}$
$\setminus \{3, 4, 9\} = \{1, 2, 5, 6, 8\}$

2. $S \times T = \{(s,t) : s \in S, t \in T\}$. But T is empty so $S \times T$ is empty.

3. b) $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$

Let $x \in S \cup (T \cap U)$. Then either $x \in S$ or $x \in T \cap U$. So either $x \in S$ or x is in both T and U. Hence x is in S or T <u>and</u> x is in S or U. Thus $x \in (S \cup T) \cap (S \cup U)$. In conclusion, $S \cup (T \cap U) \subset (S \cup T) \cap (S \cup U)$.

Now let $x \in (S \cup T) \cap (S \cup U)$. So $x \in (S \cup T)$ and $x \in (S \cup U)$.

Hence x is in S or T and x is in S or U.
Thus x is in S or x is in both T and U.
In conclusion $x \in S \cup (T \cap U)$. We see that
$(S \cup T) \cap (S \cup U) \subset S \cup (T \cap U)$.

All together, $S \cup (T \cap U) = (S \cup T) \cap (S \cup U)$.

d) $(S \setminus T) \cup (T \setminus S) = (S \cup T) \setminus (S \cap T)$

Let $x \in (S \setminus T) \cup (T \setminus S)$. So $x \in S \setminus T$ or $x \in T \setminus S$.
So x is in S but no T or x is in T but not S.
Thus x is in S or T but x is not in both
T and S. In conclusion, $x \in (S \cup T) \setminus (S \cap T)$.
Thus $(S \setminus T) \cup (T \setminus S) \subset (S \cup T) \setminus (S \cap T)$.

Now if $x \in (S \cup T) \setminus (S \cap T)$ then
x is in S or T but x is not in both
S and T. Thus x is in S but not T or
x is in T but not S. Hence $x \in (S \setminus T) \cup (T \setminus S)$.
In conclusion, $(S \cup T) \setminus (S \cap T) \subset (S \setminus T) \cup (T \setminus S)$.

All together,
$(S \setminus T) \cup (T \setminus S) = (S \cup T) \setminus (S \cap T)$.

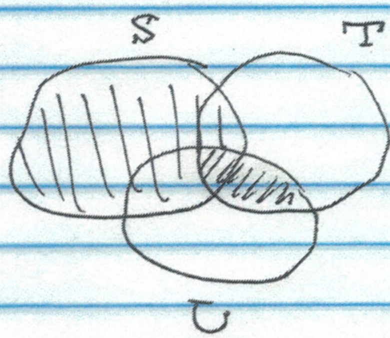
f) $S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U)$

Let $x \in S \setminus (T \cap U)$. Then $x \in S$ but x is not in both T and U. So $x \in S \setminus T$ or $x \in S \setminus U$. Therefore $x \in (S \setminus T) \cup (S \setminus U)$. Thus $S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U)$.

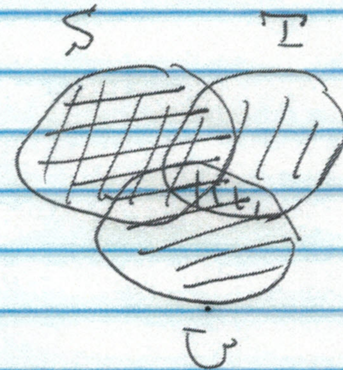
Now if $x \in (S \setminus T) \cup (S \setminus U)$, Then x is in S but not T, or x is in S but not U. Thus x is in S but not in $T \cap U$. Therefore $x \in S \setminus (T \cap U)$. In conclusion, $(S \setminus T) \cup (S \setminus U) \subset S \setminus (T \cap U)$.

All together, $S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U)$.

5 b)

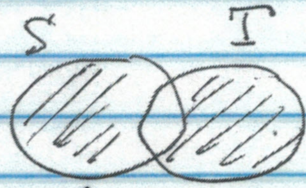


$S \cup (T \cap U)$

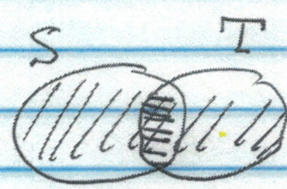


$(S \cup T) \cap (S \cup U)$

d)

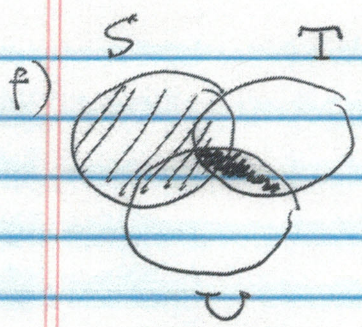


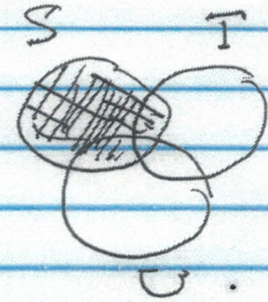
$(S \setminus T) \cup (T \setminus S)$



$(S \cup T) \setminus (S \cap T)$

f)





$S \setminus (T \cap U)$ $\qquad$ $(S \setminus T) \cup (S \setminus U)$

7. $\mathbb{Q} \setminus \mathbb{Z}$ = all rational numbers that are not integers.
$\mathbb{R} \setminus \mathbb{Q}$ = all real numbers that are not rational.

10. $S = \{a, b, 1, 2\}$

$P(S) = \{\{a\}, \{b\}, \{1\} \{2\}, \{a, b\}, \{a, 1\}, \{a, 2\}, \{b, 1\}, \{1, 2\}, \{b, 2\}, \{a, b, 1\}, \{a, b, 2\}, \{b, 1, 2\}, \{a, 1, 2\}, \{a, b, 1, 2\}, \phi\}$

11. $S = \{s_1, s_2, \dots, s_k\}$

Total no. of subsets
= # sets w/ 0 elt + # sets w/ 1 elt + # sets w/ 2 elts
+ ... + # sets w/ k elts.

$= \binom{k}{0} + \binom{k}{1} + \binom{k}{2} + \cdots + \binom{k}{k} = 2^k$.

13. Let $S_1 = \{-1, 0, 1\}$
$S_2 = \{-2, 2\}$
$S_3 = \{-3, 3\}$
$\vdots$
$S_j = \{-j, j\}$.

Then each S_j is finite but $\bigcup_j S_j = \mathbb{Z}$.

26. If $S = T$ then each subset of S is a subset of T and each subset of T is a subset of S. Hence $\wp(S) = \wp(T)$.

If $\wp(S) = \wp(T)$ then $S \in \wp(S)$ and $T \in \wp(T)$ hence $S = T$.

28. a) $S = \{a, b, c, d\}$, $T = \{1, 2, 3\}$, $U = \{b, 2\}$.
$\{a\} \subset S$ but $\{a\} \notin S$.

e) $\{a\} \subset S$ so $\{a\} \in \wp(S)$.

f) $\{a\} \subset S$, $\{a, b\} \subset S$ so $\{\{a\}, \{a, b\}\} \subset \wp(S)$.

i) $b \in S$ and $b \in U$ so $b \in S \cap U$.

n) ϕ is an element of <u>every</u> power set.