DATE

PURPOSE

MATH 310 HW5 SOLUTIONS

CUE COLUMN

NOTES

Chapter 5

1\. Let $f \in (S^T)^U$. So

$f: U \to S^T$.

Let $g(t,u) = [f(u)](t)$.

Then $g \in S^{T \times U}$.

Conversely, let $g \in S^{T \times U}$.

Then

$g: T \times U \to S$.

Let $[f(u)](t) = g(t,u)$.

Then $f \in (S^T)^U$. So the two sets have the same cardinality.

4\. Let $S = \{s\}$

$T = \{\{s\}\}$

$U = \{\{\{s\}\}\}$

$V = \{\{\{\{s\}\}\}\}$

Then $S \in T \in U \in V$.

SUMMARY

7. Let $(s,t),(s',t') \in S \times T$. Say that

$(s,t) < (s',t')$ if either

$s < s'$

or $s = s'$ and $t < t'$. This well orders $S \times T$. For if $A \subset S \times T$, choose the least element s that occurs in A. Then choose the least element t that goes with that s.

8. Any set S is a subset of itself. It cannot be an element of itself by the Axiom of Regularity.

17. You can easily choose the left shoe from each pair of shoes. This does not work with socks because there is no way to distinguish the socks in a pair.

18. If neither A nor B is empty, then let $a \in A$, $b \in B$. Then $(a,b) \in A \times B$, so $A \times B \neq \emptyset$.

22. Let $A \doteq \mathcal{P}(A)$.
Then A and $\mathcal{P}(A)$ have the same number of elements, and that is impossible.

25. The rationals and the reals are not well ordered.

27. The rationals and the reals and the integers are totally ordered but not well ordered.

Chapter 6

1. $\mathbb{Q}$ is closed under $+, -, \cdot, \div$ provided we do not divide by 0.
$\mathbb{R} \setminus \mathbb{Q}$ is not closed under any of those operations.

$(3+\sqrt{2}) + (-\sqrt{2}) = 3$ which is rational

$\sqrt{2} \cdot \sqrt{2} = 2$ which is rational

$\sqrt{3} - \sqrt{3} = 0$ which is rational

$\frac{\sqrt{2}}{\sqrt{2}} = 1$ which is rational.

3. Let $(a,b), (a',b') \in \mathbb{Z} \times \mathbb{Z}$. Say that $(a,b) \sim (a',b')$ if $a+b' = a'+b$. This is an equivalence relation. Obviously (a,b) is related to $(a-b, 0)$. So the set of equivalence classes is
$\{(z,0) : z \in \mathbb{Z}\} \cong \{z : z \in \mathbb{Z}\}$.
The set of equiv. classes is just the integers.

4. Let $x_j = q + \frac{\sqrt{2}}{j}$, $j = 1, 2, \ldots$

5. If $x = \pi + e$ and $y = \pi - e$ were both rational then

$x + y = 2\pi$ would be rational

and that is false.

6. S must be an interval

$$(a, b)$$

or

$$[a, b)$$

or

$$(a, b]$$

or

$$[a, b].$$

9. Let $(a,b) \sim (a', b')$, $(c,d) \sim (c', d')$

So $a + b' = a' + b$, $c + d' = c' + d$.

Write
$$\begin{array}{c} a + b' = a' + b \\ \underline{c' + d = c + d'} \\ (a+d) + (b'+c') = (a'+d') + (b+c) \end{array}$$

So $(a+d, b+c) \sim (a'+d', b'+c')$

This is $(a,b) - (c,d)$. This is $(a',b') - (c',d')$.