Math 310   Fall, 2020
Krantz

FIRST MIDTERM EXAM

General Instructions: Read the statement of each problem carefully. If you want full credit on a problem then you must show your work. If you only write the answer then you will *not* receive full credit.

   Be sure to ask questions if anything is unclear. This exam has 8 questions and is worth 100 points. You will have 50 minutes to take this exam.

(8 points)   **1.** Write the converse and the contrapositive of the sentence

   If dogs can fly, then chickens have lips.

   Label each one.

(8 points)   **2.** Are the statements $(\mathbf{A} \Rightarrow\, \sim \mathbf{B})$ and $(\sim \mathbf{A} \vee \sim \mathbf{B})$ logically equivalent? Why or why not?

(10 points)   **3.** Express the statement $\exists x, \sim \mathbf{P}(x)$ using $\forall$ instead of $\exists$.

(8 points)   **4.** Prove that the product of an even integer and an odd integer is even.

(10 points)   **5.** Prove that the integer 5 does not have a rational square root.

(8 points) **6.** Use mathematical induction to prove that

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

for any positive integer n.

(8 points) **7.** Give a truth table for the statement $(\mathbf{A} \wedge \sim \mathbf{B}) \Rightarrow (\sim \mathbf{A} \vee \mathbf{B})$.

(8 points)   **8.** Use any method to prove the pigeon-hole principle.

(8 points)   **9.** Let S and T be sets. Prove that

$$(S \setminus T) \cup (T \setminus S) = (S \cup T) \setminus (S \cap T).$$

(9 points) **10.** Let $S = \{1, 2, 3, 4, 5\}$, $T = \{3, 4, 5, 7, 8, 9\}$, $U = \{1, 2, 3, 4, 9\}$. Calculate

       (a) $(S \cap T) \cup U$

       (b) $(S \cap U) \cup T$

       (c) $(S \cup T) \setminus (S \cap T)$

(8 points) **11.** Let S, T, and U be sets. Draw two Venn diagrams to illustrate the identity

$$S \setminus (T \cup U) = (S \setminus T) \cap (S \setminus U).$$

(7 points) **12.** Give an explicit description of the power set of $A = \{1, 2, a, b\}$.