PRACTICE EXAM FOR FIRST MIDTERM

(8 points)   **1.** Write the converse and the contrapositive of the sentence

If down is up, then life goes on.

Label each one.

(8 points) **2.** Are the statements $\mathbf{A} \vee \mathbf{B}$ and $\sim \mathbf{A} \wedge \mathbf{B}$ logically equivalent? Why or why not?

(10 points) **3.** Express the statement $\forall x, \sim \mathbf{P}(x)$ using $\exists$ instead of $\forall$.

(8 points)   **4.** Prove that the square of an odd integer is odd.

(10 points)   **5.** Prove that the integer 6 does not have a rational square root.

(8 points) **6.** Use mathematical induction to prove that the sum of the first n odd integers is n^2.

(8 points) **7.** Give a truth table for the statement $(\mathbf{A} \vee \sim \mathbf{B}) \Rightarrow (\sim \mathbf{A} \wedge \mathbf{B})$.

(8 points)   **8.** Use any method to prove that $2^k > 1 + 2k$ for $k > 2$.

(8 points)   **9.** Let S, T, and U be sets. Prove that

$$S \setminus (T \cap U) = (S \setminus T) \cup (S \setminus U).$$

(9 points) **10.** Let $S = \{a, b, c, d, e\}$, $T = \{b, d, f, h\}$, and $U = \{a, d, g\}$. Calculate

      (a) $(S \setminus T) \cup U$

      (b) $(S \cap T) \setminus U$

      (c) $(S \cup T) \cap U$

(8 points) **11.** Let S, T, and U be sets. Draw two Venn diagrams to illustrate the identity

$$(S \setminus T) \cup (T \setminus S) = (S \cup T) \setminus (S \cap T).$$

(7 points)   **12.** Calculate the power set of $\{1, @, *, \gamma\}$.