Math 470 April 16, 2012
Spring, 2012

Final Exam

General Instructions

Read each question carefully. Answer just what is asked; nothing more and nothing less. Write out your answers completely, using complete sentences where appropriate. You can write out your solutions on ordinary $8.5'' \times 11''$ paper.

You can consult any book in the library, and you can consult the Internet. The only *person* whom you may consult is me (S. G. Krantz), and I encourage you to do so.

The point value of each problem is specified. The total number of points is 200.

This exam is due on Tuesday, May 8, 2012 at noon. You can put it in an envelope in the basket on my office door (Room 103, Cupples I). Or you can scan it in and send it to me as an e-mail attachment.

1. (15 points) How many distinct cycles are there in the complete graph on n vertices?

2. (15 points) Given a graph G, prove that the symmetric difference of two even subgraphs is an even subgraph. Is the assertion still true if "even" is replaced by "odd"?

3. (15 points) Prove that $K_{3,3}$ and $(K_{3,2} + K_3)$ are not isomorphic by using spectral theory.

4. (15 points) Given that $\phi(G; \lambda) = \lambda^8 - 24\lambda^6 - 64\lambda^5 - 48\lambda^4$, determine the graph G completely.

5. (10 points) Compute the expected number of fixed points in a random permutation of $[n]$.

6. (15 points) Determine the expected number of vertices of degree k in a random n-vertex graph with edge probability p.

7. (15 points) Determine the expected number of monochromatic triangles in a random 2-coloring of $E(K_6)$.

8. (15 points) Prove that a graph with n vertices and average degree $d \geq 1$ has an independent set with at least $n/(2d)$ vertices. [**Hint:** Choose the right probability to assign to each vertex.]

9. (10 points) Every tree is bipartite. But not every bipartite graph is a tree. Explain in detail.

10. (15 points) Let T be a tree with average degree a. In terms of a, determine $n(T)$.

11. (10 points) Prove that every n-vertex graph with m edges has at least $m - n + 1$ cycles.

12. (10 points) **TRUE or FALSE:** Every graph with fewer edges than vertices has a component that is a tree.

13. (10 points) Prove that a graph G is a tree if and only if G is connected and every edge is a cut-edge.

14. (15 points) For each $k = 3, 4, 5, 6$, list the isomorphism classes of trees with maximum degree k and at most six vertices.

15. (15 points) For $2 \le k \le n - 1$, prove that the n-vertex graph formed by adding one vertex adjacent to every vertex of P_{n-1} has a spanning tree with diameter k.