# Ma 4111: Advanced Calculus
# Homework Assignment 7

Prof. Wickerhauser

Due Thursday, December 6th, 2012

Please return your solutions to the instructor by the end of class on the due date. You may collaborate on these problems but you must write up your own solutions. **Late homework will not be accepted.**

1. Let $a_n = \frac{n!}{n^n}$ for $n > 0$. Prove that $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1/e$.
2. Suppose that $\{a_n\}$ is a bounded sequence of real numbers with the property that $\liminf_{n\to\infty} a_n \geq \limsup_{n\to\infty} a_n$. Prove that $\lim_{n\to\infty} a_n$ exists.
3. Suppose that $\sum_{n=1}^{\infty} a_n$ converges absolutely. Define $\{b_n\}$ by
$$b_n = \begin{cases} a_n^2, & \text{if } n \text{ is even,} \\ -a_n^n, & \text{if } n \text{ is odd.} \end{cases}$$
Prove that $\sum_{n=1}^{\infty} b_n$ converges absolutely.
4. Determine with proof whether the following series converges:
$$\sum_{n=1}^{\infty} \left(\sqrt{1+n^6} - n^3\right)$$
5. Determine with proof whether the following series converges:
$$\sum_{n=1}^{\infty} (\sqrt{1+n} - \sqrt{n})$$
6. Determine with proof whether the following series converges:
$$\sum_{n=2}^{\infty} (\log n)^{-\log n}$$
7. Find a double sequence $\{a_{n,m}\}$ such that $\lim_{n\to\infty} a_{n,m} = 0$ for all fixed $m$ and $\lim_{m\to\infty} a_{n,m} = 0$ for all fixed $n$, but $\lim_{n,m\to\infty} a_{n,m}$ does not exist.
8. Find the Cesàro sum of the complex-valued series $\sum_{n=0}^{\infty} i^n$, where $i^2 = -1$.
9. Prove that $\prod_{n=2}^{\infty}(1 - n^{-2})$ converges and evaluate it.
10. Prove that if a double series converges absolutely, then it converges.