Ma 416: Complex Variables
Final Examination

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You may use your textbook and graded homework sets as well as any calculator. Please write your complete answers in the bluebook.

1. Use Rouché's theorem to determine the number of zeros of $2e^{z/3} + z$ satisfying $|z| < 1$.

2. Find the Laurent series for $f(z) = e^{-z^2}/(1 + z^2)$ valid in a punctured neighborhood of ∞.

3. Prove that an entire function whose imaginary part is bounded must be constant. (Hint: apply Liouville's theorem to the function e^{if}.)

4. Suppose f is analytic on the closed unit disk, $f(0) = 0$, and $|f(z)| \leq 17$ whenever $|z| = 1$. How big can $f((1+i)/2)$ be?

5. Show that
$$\frac{1}{1} + \frac{1}{2} - \frac{1}{3} - \frac{1}{4} + \frac{1}{5} + \frac{1}{6} - \frac{1}{7} - \frac{1}{8} + \cdots$$
converges.

6. Verify that $1/(1 - z)$ can be continued outside the unit disk by expanding it about $z = 3i$.

7. (a) Is the function $h(x, y) = x^3 - 3xy^2$ the real part of some function $f(x+iy)$ analytic in the open unit disk $D \subset \mathbf{C}$?

   (b) Is the function $g(x, y) = x^3 + 3xy^2$ the imaginary part of some function $f(x + iy)$ analytic in the open unit disk $D \subset \mathbf{C}$?

8. Find a Möbius transform mapping $0, 1, \infty$ to $1, 2, 3$, respectively.