Ma 416: Complex Variables
Homework Assignment 2

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Due Thursday, September 15th, 2005

1. Prove or find a counterexample to the following statements:
   (a) If $f(x) = O(g(x))$ as $x \to 0$, then $f(x)/g(x) \to 0$ as $x \to 0$.
   (b) If $f(x) = o(g(x))$ as $x \to \infty$, then $f(x)/[1 + |g(x)|] \to 0$ as $x \to \infty$.
   (c) If $f(x) = o(g(x))$ as $x \to 1$, then $f(x) = O(g(x))$ as $x \to 1$.
   (d) If $f(x) = o(x)$ as $x \to 0$, then $f(x) = O(x^2)$ as $x \to 0$.

2. Let $f(x, y) = u(x, y) + iv(x, y)$ be a complex-valued function of two real variables. Write $z = x + iy$ for the complex variable with real part x and imaginary part y. Show that the Cauchy-Riemann equations are equivalent to the equation
$$\frac{\partial}{\partial \bar{z}} f(z) = 0,$$
using the defintion $\frac{\partial}{\partial \bar{z}} = \frac{1}{2}\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right]$ on page 19 of our textbook.

3. Determine whether the following functions $f(z) = f(x + iy)$ are analytic:
   (a) $f(z) = x^2 + y^2$
   (b) $f(z) = x^2 - y^2$
   (c) $f(z) = x^2 - y^2 + 2ixy$

4. Find the domain of convergence of the following power series:
$$\text{(a)} \sum_{n=0}^{\infty} (z - 3i)^{2n} \qquad \text{(b)} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}}$$

5. Write a power series for the k^{th} derivative of
$$\sum_{n=0}^{\infty} (-1)^n z^n,$$
for all $k = 1, 2, \ldots$, and determine the domain of convergence. What functions do these power series represent?

6. Determine, with proof, whether the following series converge uniformly on the domain $|z| < 1$:
$$\text{(a)} \sum_{n=1}^{\infty} \frac{z^n}{n^2}; \qquad \text{(b)} \sum_{n=0}^{\infty} z^n.$$