Ma 416: Complex Variables
Homework Assignment 7

Prof. Wickerhauser

Due Thursday, October 27, 2005

Read R. P. Boas, *Invitation to Complex Analysis*, Chapter 2, sections 12A–13C.

1. Use the argument principle to count the zeros of $P(z) = z^4 + z^3 + 6z^2 + 3z + 5$ in the left half-plane $\{\Re z < 0\}$ and right half-plane $\{\Re z > 0\}$ of the complex plane.

2. Use Rouché's theorem to determine the number of zeros of $3e^{z/2}+z$ satisfying $|z| < 1$.

3. Suppose $\{f_n : n = 1, 2, \ldots\}$ is an infinite sequence of analytic functions that converges uniformly in all compact subsets of a region D containing 0.

   (a) Show that $\{\exp(f_n) : n = 1, 2, \ldots\}$ is also an infinite sequence of analytic functions that converges uniformly in each compact subset of D.

   (b) Show that if $\lim_{n\to\infty} \exp(f_n(0)) = 0$, then $\lim_{n\to\infty} \exp(f_n(z)) = 0$ for all $z \in D$.

4. Is it possible for a function $f = f(z)$ which takes only purely imaginary values to be analytic on $\{|z| < 1\}$?

5. Show that $f(z) = z/(1 - z)^2$ is univalent in $|z| < 1$.

6. Prove that the converse to Darboux's theorem is false: Find a simple closed curve S and an analytic function $f = f(z)$ such that f is univalent inside S but not univalent on S.