# Ma 416: Complex Variables
# Homework Assignment 10

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Due Thursday, November 17th, 2005

Read R. P. Boas, *Invitation to Complex Analysis*, Chapter 3, sections 17A–18C.

1. Verify that $1/(1-z)$ can be continued outside the unit disk by expanding it about $z = ih$ for some $0 < h < 1$. Can you find an expansion about $z = i$?

2. Suppose $f(z) = \sum_{n=0}^{\infty} z^{2^n}$. Find the radius of convergence $R$ of this power series. Is there a function $g(z)$ analytic on a larger region than $D = \{|z| < R\}$ that agrees with $f(z)$ at all $z \in R$?

3. Use Abel's theorem to conclude that $\sum_{n=1}^{\infty}(-1)^n/n = -\ln 2$.

4. Show that
$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{8}} + \cdots$$
converges.

5. Find the $(C, 1)$ sums of the series

   (a) $\sum_{n=0}^{\infty}(-1)^n$,

   (b) $\sum_{n=1}^{\infty}(-1)^n$, and

   (c) $1 - 1 + 0 + 1 - 1 + 0 + 1 - 1 + 0 + \cdots$ (where the terms $1, -1, 0$ repeat forever).

6. Show that if $f(x) = \sum_{n=0}^{\infty} a_n x^n$ is convergent in $|x| < 1$ with $|na_n| \leq 8$ for all $n$, and $f(x) \to +\infty$ as $x \to 1-$, then $\sum_{n=0}^{\infty} a_n = +\infty$.