Ma 449: Numerical Applied Mathematics. Final Examination.

Prof. Wickerhauser; 6:00–8:00pm Friday, December 12th, 2008

You may use a calculator and the textbook. Please write your answers in the bluebook.

1. Suppose that $Q(h) = Q(f, [a, b], h)$ is a quadrature rule that satisfies
$$Q(h) = \int_a^b f(x)\,dx + O(h^3).$$
Find a formula that combines $Q(h)$ and $Q(h/2)$ to give $\int_a^b f(x)\,dx + O(h^4)$.

2. (a) Use the composite trapezoid rule with stepsize $h = 1$ to approximate $\displaystyle\int_{-1}^{1} \frac{dx}{1+x^2}$.

   (b) Estimate an upper bound for the error.

3. Use the two-point Gauss-Legendre integration rule to approximate $\displaystyle\int_{-1}^{1} \frac{dx}{1+x^2}$.

4. The function $f(x) = \exp(x) + 4\cos(x)$ is unimodal on the interval $[1, 2]$. Find its minimum in that interval to 4 significant digits.

5. Use Euler's method with step size $h = 1$ to solve the following initial value problem on the interval $[0, 3]$:
$$y'(t) = (3 - 2t)y(t), \quad 0 < t < 3; \qquad y(0) = 1.$$
Tabulate the approximations $y(1)$, $y(2)$, and $y(3)$, and compare the resulting $y(3)$ with the exact value $y(3) = 1$.

6. Use the finite differences method with step size $h = 1$ to solve the following boundary value problem on the interval $[0, 3]$:
$$x''(t) = (4t^2 - 12t + 11)x(t); \qquad x(0) = 1, \quad x(3) = 1,$$

7. Let $A = \begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}$.

   (a) Find the eigenvalues of A.

   (b) Find a Givens rotation matrix $G = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, for some θ, such that GA is upper-triangular.

   (c) Find the eigenvalues of GAG^T.