Ma 449: Numerical Applied Mathematics. Final Examination.

Prof. Wickerhauser; 6:00–8:00pm Friday, December 11th, 2009

You may use a calculator and the textbook with any notes you wrote in the textbook or on an 8.5 by 11 inch sheet of paper. Please write your answers in the bluebook.

1. Suppose f''' is continuous and bounded by M_3. Use Taylor's theorem to estimate the error in the difference formula $f'(x) \approx [f(x+h/2) - f(x-h/2)]/h$, in terms of M_3, as $h \to 0$.

2. Suppose that $Q(h) = Q(f, [a,b], h)$ is a quadrature rule that satisfies

$$Q(h) = \int_a^b f(x)\,dx + O(h^4),$$

depends smoothly on h, and is an even function of h: $Q(-h) = Q(h)$ for all h. Find a formula that combines $Q(h)$ and $Q(h/2)$ to give $\int_a^b f(x)\,dx + O(h^6)$.

3. (a) Use the composite trapezoid rule with stepsize $h = 1$ to approximate $\displaystyle\int_{-1}^{1} e^{x^2}\,dx$.

   (b) Estimate an upper bound for the error.

4. Use the four-point Gauss-Legendre quadrature rule to approximate $\displaystyle\int_{-1}^{1} e^{x^2}\,dx$.

5. The function $f(x) = \log(x) + \exp(x) - 7x$ is unimodal on the interval $[1,3]$. Find its minimum in that interval to 4 significant digits.

6. Solve the following initial value problem on the interval $[0,1]$:

$$y'(t) = (1-2t)y(t), \quad 0 < t < 1; \qquad y(0) = 1.$$

Use Heun's method with a step size $h = 1$.

7. Use the finite differences method with step size $h = 1$ to solve the following boundary value problem on the interval $[0,3]$:

$$x''(t) = (1+t)x(t); \qquad x(0) = 1, \quad x(3) = 1,$$