Ma 449: Numerical Applied Mathematics. Final Examination.

Prof. Wickerhauser; 6:00–8:00pm Friday, December 16th, 2010

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You may use a calculator and refer to the textbook and any notes you wrote in the textbook. No other materials are permitted. Please write your answers in the bluebook.

1. Suppose f'' is continuous and $|f''| \leq M_2$. Use Taylor's theorem to estimate the error in the difference formula $f'(x) \approx [f(x+2h) - f(x)]/2h$, in terms of M_2, as $h \to 0$.

2. Suppose that $Q(h) = Q(f, [a,b], h)$ is a quadrature rule that satisfies

$$Q(h) = \int_a^b f(x)\,dx + O(h^2),$$

depends smoothly on h, and is an even function of h: $Q(-h) = Q(h)$ for all h. Given the table of values below for a particular function f, use Romberg integration to find $\int_a^b f(x)\,dx$ to order $O(h^6)$:

$Q(h)$	1.20000	
$Q(h/2)$	1.23000	
$Q(h/4)$	1.24500	

3. Determine, with proof, the degree of precision of the quadrature rule

$$Q(f, [0,1]) \stackrel{\text{def}}{=} \frac{1}{2}f(x_1) + \frac{1}{2}f(x_2), \qquad x_1 = \frac{1}{2} - \frac{1}{\sqrt{12}}; \quad x_2 = \frac{1}{2} + \frac{1}{\sqrt{12}}.$$

Thus the weights are $w_1 = 1/2$ and $w_2 = 1/2$. Note that the interval is $[0,1]$, not $[-1,1]$.

4. The function $f(x,y) = x^2 + x + e^{2y} - 4e^y + 17$ has a unique minimum in the x, y plane.

(a) Find the minimum (x,y) to 4 significant digits in both x and y.

(b) Starting with the initial simplex $(0,0)$, $(0,1)$, $(1,0)$, perform one step of the Nelder-Mead algorithm to find the next approximating simplex.

5. Solve the following initial value problem on the interval $[0,1]$:

$$y'(t) = (1+2t)y(t), \quad 0 < t < 1; \qquad y(0) = 1.$$

Use the fourth-order Runge-Kutta method (RK4) with a step size $h = 1$.

6. Consider the following boundary value problem on the interval $[0,3]$:

$$x''(t) = (1+2t)x(t); \qquad x(0) = 1, \quad x(3) = 1,$$

Find approximate solutions for $x(1)$ and $x(2)$ by each of the following methods:

(a) the finite differences method with step size $h = 1$;

(b) linear shooting with Euler's method and step size $h = 1$.