Math 449: Numerical Applied Mathematics
Midterm Examination

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You may use a calculator and the textbook. Please write your answers in the bluebook.

Problem 1. Express $1/7 = 0.\overline{142857}$ (base 10) in base 2 notation, giving at least 10 digits after the radix point.

Problem 2. Note: "log" means the natural logarithm.

(a) Find a polynomial $p = p(h)$ of minimal degree in h such that $\log(1 + h) = p(h) + O(h^5)$ as $h \to 0$.

(b) Find $\epsilon > 0$ such that $|\log(1 + h) - p(h)| < 0.00007$ whenever $|h| < \epsilon$.

Problem 3. The function $f(x) = e^x + \log x$ has a unique root in the interval $0 < x < 1$.

(a) Find the Newton–Raphson iteration formula for the equation $f(x) = 0$.

(b) Solve for x in $f(x) = 0$ by any method that you choose. Give at least 5 correct digits after the decimal point.

For the following four problems, let $A = \begin{pmatrix} 2 & -2 & 0 \\ -1 & 3 & -2 \\ 0 & -1 & 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, and $\mathbf{x} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$.

Problem 4. Find a factorization $A = LU$, where matrix L is unit lower triangular and matrix U is upper triangular. (Hint: no interchanges are needed.)

Problem 5. Find the determinant $\det A$.

Problem 6. Let L and U be the matrices from problem 4. Suppose that $\mathbf{y}$ and $\mathbf{z}$ are vectors in $\mathbf{R}^3$, $\mathbf{y}$ solves $L\mathbf{y} = 2\mathbf{x}$, and $\mathbf{z}$ solves $U\mathbf{z} = 3\mathbf{y}$. Compute $A\mathbf{z}$.

Problem 7. (a) Compute the vector $A\mathbf{b}$.

(b) Compute the inner product of $A\mathbf{b}$ with $\mathbf{b}$.

(c) Find the cosine of the angle between $A\mathbf{b}$ and $\mathbf{b}$

Problem 8. Find the complex exponential Fourier series for the function $f(x) = \cos(x)+\sin(2x)$. (Hint: it has finitely many nonzero terms.)

Problem 9. Fix $c > 0$ and let $P_0 = (-1, 0)$, $P_1 = (0, c)$, and $P_2 = (1, 0)$ be three points in the (x, y)-plane. Let $B = B(t)$, $0 \le t \le 1$, be the Bézier curve with control points P_0, P_1, P_2. Find the maximum y-coordinate of $B(t)$ for any t.

Problem 10. Find the least squares curve of the form $y(x) = Ax + B$ for the three points $(0, 0)$, $(0, 1)$, and $(2, 1)$.