Ma 449: Numerical Applied Mathematics
Final Examination.

Professor Wickerhauser

Due by **8:00 pm Thursday, December 12th, 2019**

5 problems on 2 pages

You may use a calculator or computer and refer to your class notes, the textbook, its website, and the Math 449 web site. No collaboration with any other person is allowed.

Please include any computer input and output that you used to solve a problem.

1. (20 points)

   Let $Q(h)$ be the composite trapezoid quadrature rule approximation with stepsize h to the following integral:

   $$\int_0^1 \exp(\cos(t))\, dt$$

   (a) Fill in this table:

$Q(0.004)$	
$Q(0.002)$	
$Q(0.001)$	

   (b) Using the values from the table in part a, compute an approximation to the integral using Richardson extrapolation.

   (c) Using the values from part a, estimate the absolute error in $Q(0.001)$.

2. (20 points)

   Suppose $f = f(x)$ has continuous derivatives of all orders satisfying $|d^k f(x)/dx^k| \leq M_k$ for all real x and all $k = 1, 2, 3, \ldots$, where M_k is a known positive constant for each k. Use Taylor's theorem to estimate the error in the difference formula

   $$f'(x) \approx \frac{f(x + h/2) - f(x - h/2)}{h}$$

   in terms of M_k and h.

3. (20 points)

   The function $f(x, y) = 4e^{2x} - 5e^x + 7ye^x + 6y^2 + 2$ has a unique minimum in the x, y plane.

   (a) Starting with the initial simplex $(0, 0)$, $(0, 1)$, $(1, 0)$, perform one step of the Nelder-Mead algorithm to find the next approximating simplex.

   (b) Use the Nelder-Mead algorithm from the textbook to find the minimum (x, y) to 4 significant digits in both x and y.

   (c) Set $\nabla f(x, y) = (0, 0)$ and solve for the exact value of the minimum. Does it agree with the results of part (b)?

4. (20 points)

   Consider the following initial value problem on the interval $[0, 1]$:

   $$y'(t) = \cos(t)(y(t) - \sin(t)), \quad 0 < t < 1; \qquad y(0) = 2.$$

   Use Heun's method and choose a step size small enough to give a 7 significant digit approximation to $y(1)$, namely a final global error less than 5×10^{-7}.

5. (20 points)

   Consider the following boundary value problem on the interval $[0, 1]$:

   $$x''(t) = \sin(t)x'(t) + \exp(t)x(t) + \cos(t); \qquad x(0) = 1, \quad x(1) = 2,$$

   Find approximate solutions for $x(0.5)$ by each of the following methods:

   (a) linear shooting with the 4th order Runge-Kutta method and step sizes $h = 0.02$ and $h = 0.01$ (50 and 100 steps);

   (b) finite differences with step sizes $h = 0.002$ and $h = 0.001$ (500 and 1000 steps).

   (c) Estimate the error in $x(0.5)$ with the smaller step size for both methods, using the results from parts (a) and (b). Which solution is more accurate?