Math 449: Numerical Applied Mathematics
Midterm Examination

Professor Wickerhauser

Wednesday, 19 October 2016

6 problems on 1+6 pages

No written material, no collaboration, and no electronic devices are allowed. Please write your answers in the space provided. You have 50 minutes.

Useful formulas:

- The Bézier curve through control points $P_0, \ldots, P_n$ is $B(t) = P_0 B_{0,n}(t) + \cdots + P_n B_{n,n}(t)$, $0 \le t \le 1$, where $B_{k,n}(t) = \begin{pmatrix} n \\ k \end{pmatrix} t^k (1-t)^{n-k}$.

- **Theorem.** If there exists $K < 1$ such that $|g(x) - g(y)| \le K|x-y|$ for all x, y, then g is a contraction map and iteration $x_{n+1} = g(x_n)$ from any initial point x_0 will converge to a unique limit p satisfying

$$|x_n - p| \le |x_1 - x_0| K^n / (1-K).$$

- **Theorem.** If f has $n+1$ continuous derivatives, then for any a, x there exists a point c betwen a and x such that

$$f(a+x) = f(a) + xf'(a)/1! + \cdots + x^n f^{(n)}(a)/n! + x^{n+1} f^{(n+1)}(c)/(n+1)!$$

- Normal equations for the least squares line $y = Ax + B$ determined by the points $\{(x_k, y_k) : k = 1, \ldots, n\}$ are

$$\begin{pmatrix} \sum x_k^2 & \sum x_k \\ \sum x_k & n \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} \sum x_k y_k \\ \sum y_k \end{pmatrix}.$$

- Newton's form for the Lagrange polynomial through the points $\{(x_k, y_k) : k = 0, 1, \ldots, n\}$ is

$$p(x) = c_0 + c_1(x - x_0) + \cdots + c_n(x - x_0) \cdots (x - x_{n-1}),$$

where $c_0 = y_0, c_1 = dy_0, \ldots, c_n = d^n y_0$ is the diagonal of the divided difference table

k	x	y	dy	d^2y	$\cdots$
0	x_0	y_0			
1	x_1	y_1	$dy_0 = (y_1 - y_0)/(x_1 - x_0)$		
2	x_2	y_2	$dy_1 = (y_2 - y_1)/(x_2 - x_1)$	$d^2y_0 = (dy_1 - dy_0)/(x_2 - x_0)$	
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$