Math 450: Mathematics for Multimedia Midterm Examination

Name: ______________________________

Wednesday, 8 March 2017

7 problems on 7 pages

Use only this test and a pen or pencil. Please write your complete answers in the space provided. You have 50 minutes.

1. Give a 3 hexadecimal digit approximation to $1/3$.

2. (a) How many integers in the set $\{0, 1, \dots, 34\}$ are relatively prime with 35?

   (b) Find an integer x such that $14x \equiv 1 \pmod{35}$ or prove that none exists.

3. Suppose that $\mathbf{x}, \mathbf{y}$ are vectors in an inner product space $\mathbf{X}$, with $\|\mathbf{y}\| = 5$ and $\|\mathbf{x}\| = 2$.

   (a) What is the minimum possible value of $\langle \mathbf{x}, \mathbf{y} \rangle$?

   (b) What is the minimum possible value of $\|\mathbf{x} + \mathbf{y}\|$?

4. Suppose that $B = \{b_1, b_2, b_3\}$ is the basis of the inner product space $\mathbf{R}^3$ consisting of the vectors

$$b_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

   Let $B' = \{b_1', b_2', b_3'\}$ be the biorthogonal dual basis to B.

   (a) Show that $b_1 \in \operatorname{span}\{b_2', b_3'\}^\perp$.

   (b) Find scalars c_1, c_2, c_3 such that

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} = c_1 b_1' + c_2 b_2' + c_3 b_3',$$

   or prove that no such scalars exist. (Hint: it is not necessary to find b_1', b_2', b_3'!)

5. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation defined by $T(x, y) = (x + y, x - y)$. Compute $\|T\|_{\text{op}}$ with respect to the usual Euclidean norms.

6. Suppose that the complex exponential Fourier series of the 1-periodic function

$$f(t) \stackrel{\text{def}}{=} \sum_{n=-\infty}^{\infty} c(n)e^{2\pi int}$$

has coefficients $c(n) = 0$ for $n < 0$ and $c(n) = (1/2)^n$ for $n \geq 0$. Find $f(1/2)$.

7. Suppose that $f = f(x)$ has Fourier integral transform

$$g(\xi) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(x)e^{-2\pi ix\xi}\, dx.$$

Compute the Fourier integral transform of $3f(2x - 1)$ in terms of g.