Ma 541: Topics in Applied Mathematics: Wavelet Algorithms
Homework Assignment 1

Prof. Wickerhauser

Due Friday, September 12th, 2008

1. Suppose that $B = \{e_n : n = 1, \ldots, N\}$ and $B' = \{e'_n : n = 1, \ldots, N'\}$ are orthonormal bases for a Hilbert space H. Show that $N = N'$.

2. Show that an orthonormal basis B for a Hilbert space H is a *Schauder basis*: two equal expansions
$$x = \sum_{b\in B} c_b\, b = \sum_{b\in B} c'_b\, b \in H,$$
must have equal expansion coefficients $c'_b = c_b$ for all $b \in B$. (Hint: use Parseval's formula.)

3. Let B be the Haar wavelet basis for $L^2([0,1])$. Write an R program to compute a specified expansion coefficients of a specified function $f : [0,1] \to \mathbf{R}$.

   Use your program to compute the inner product $\langle w_{3,3}, f\rangle$ for $f(x) = \sin(8\pi x)$.

4. Fix
$$\theta(t) \stackrel{\text{def}}{=} \begin{cases} \frac{\pi}{4}\sin(\frac{3t}{2}), & \text{if } -\frac{\pi}{3} \le t \le \frac{\pi}{3}; \\ -\frac{\pi}{4}, & \text{if } t < -\frac{\pi}{3}; \\ \frac{\pi}{4}, & \text{if } t > \frac{\pi}{3}. \end{cases}$$
Note that θ has one continuous derivative on $\mathbf{R}$.

   (a) Plot the modulus of $\mathcal{F}\psi(\xi)$ for the corresponding Yves Meyer wavelet ψ, on the interval $\xi \in [-10, 10]$.

   (b) Plot the Yves Meyer wavelet $\psi(x)$ on the interval $x \in [-10, 10]$.

5. Plot the following discrete cosine transform function:
$$\cos\left(\frac{\pi(m+\frac{1}{2})(n+\frac{1}{2})}{N}\right), \qquad n = 0, 1, \ldots, N-1,$$
for $N = 512$ and various values of $m \in \{0, 1, \ldots, 511\}$.